E&M waves
Polarization conventions

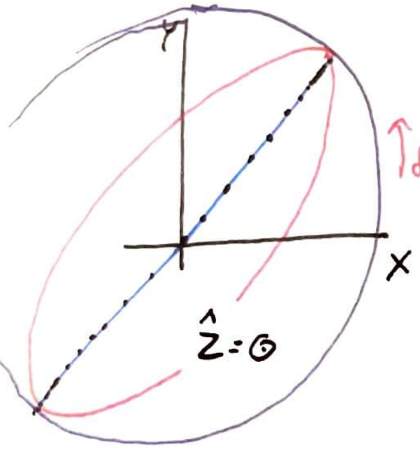
Definition of Terms

E field plane wave can be written

$$\vec{E} = (E_x \hat{x} e^{i\phi_x} + E_y \hat{y} e^{i\phi_y}) \exp(i(k \cdot \hat{z} - \omega t))$$

at fixed z (or fixed t) $\vec{E}$ traces out an ellipse.

Relative phase $\delta \equiv \phi_x - \phi_y$ $\xrightarrow[\text{sets}]{+E \text{ vector}}$ the shape



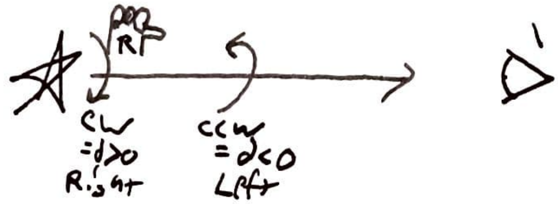
$\phi_x = \phi_y = 45^\circ$ Line

$\uparrow \delta > 0$ $\phi_x = 0$, $\phi_y = 90$ circle

$\hookrightarrow$ clockwise from observer

$E_x = E_y$

IAU pol convention



In general

E field amp also matters

Polarization usually measured in stokes parameters

$$I = \langle E_x^2 + E_y^2 \rangle / R_0$$

$$Q = \langle E_x^2 - E_y^2 \rangle / R_0$$

$$U = \langle 2 E_x E_y \cos\delta \rangle / R_0$$

$$V = \langle 2 E_x E_y \sin\delta \rangle / R_0$$

- $R_0 = \frac{4\pi}{c}$
- Avg over 1 cycle $\tau \gg (1/\Delta\nu 2\pi)$
- assuming stochastic source

EM cont

Def: polarized flux

$$I_p = \sqrt{(Q^2 + U^2 + V^2)}$$

polarization fraction

$$P = \frac{I_p}{I}$$

Note that stokes parameters are also definable in terms of circular polarizations.

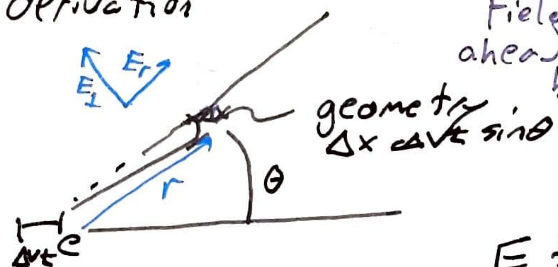
Antennas

ERA 2.7
Condon + Ransom

~~Moving~~ e^-
Accelerating e^-'s generate propagating E fields

The Larmor derivation

Field lines get ahead of where they would be before acceleration by Δx



Geometry	E field
$\frac{E_\perp}{E_r} = \frac{\Delta x}{r}$	$E_r = \frac{q}{r^2}$

$$= \frac{\Delta v\, t \sin\theta}{c\,\Delta t}$$

$t = \frac{r}{c}$

+

$\frac{\Delta v}{\Delta t} \to \dot{v}$

$$\boxed{E_\perp = \frac{q\dot{v}\sin\theta}{rc^2}}$$

power pattern → poynting flux

$$\vec{S} = \frac{c}{4\pi}\vec{E}\times\vec{B}$$

in CGS $|\vec{E}| = |\vec{B}|$

$$S = \left(\frac{q^2\dot{v}^2}{4\pi c^3}\right)\frac{\sin^2\theta}{r^2}$$


Hmm Donut.

Integrate over sphere

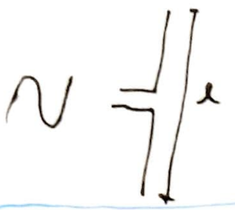
. . . . (two r^2 in dA cancel)

Note, do not use if
- $v \sim c$
- $\Delta r \lesssim$ bohr radius

$$P = \frac{2}{3}\frac{q^2\dot{v}^2}{c^3}$$

More Antennas

A dipole



No e's are not moving @ v=c. see ex in ERA showing for ~1mm wire @ 1A $V \sim .01 \frac{mm}{s}$!

Drive the dipole w/ a sinusoid

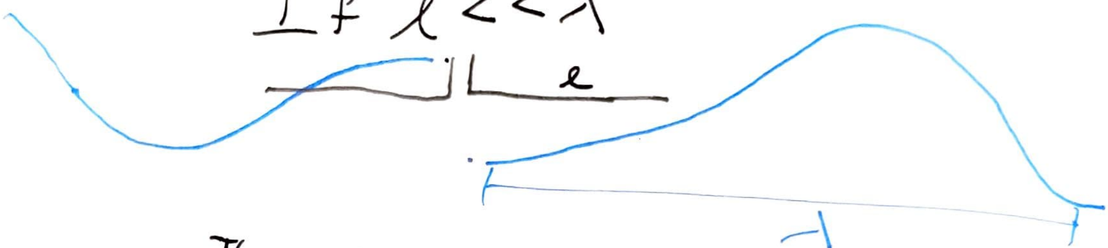
$$I = I_0 e^{-i\omega t}$$

The moving charges radiate according to Larmor.

dz an infinitesimal charge moves

$$dE_\perp = \frac{dq\,\dot{v}\sin\theta}{rc^2}$$

If $l << \lambda$



Then all dz charges are moving in phase. + $E_\perp$s will add up

$$E_\perp = \int \frac{\dot{v}\sin\theta}{rc^2}\,dq$$

$$= \int_{-l/2}^{l/2} \frac{dq}{dz}\,dz\,\frac{\dot{v}\sin\theta}{rc^2}$$

@ $r >> l$, the 'far' field r can be same for all dq.

Use our sinusoid which has $\dot{v} = -i\omega v$

$$E_\perp = \frac{-i\omega\sin\theta}{rc^2}\int \underbrace{\frac{dq}{dz}v}\,dz$$

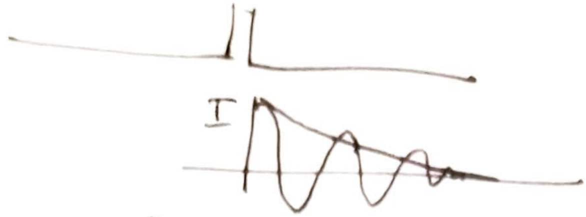
$$\frac{dq}{dz}\frac{dz}{dt} = I$$

$$E_\perp = \frac{-i\omega\sin\theta}{rc^2}\int I\,dz$$

Current is going in, not coming out

Must go to zero @ end!

Ants x3



Condon + Ransom say
current "declines almost linearly" bc it is the
"tail end of a standing wave sinusoid"

Maybe they mean sin(kz)
where $k = \frac{2\pi}{\lambda}$. if $z \ll \lambda$ then

$$\sin(kz) \approx \left(1 + \frac{kz}{2}\right)$$

But not quite.... anyway, Jim to Scott
Let

$$I(z) \simeq I_0 e^{-i\omega t}\left[1 - \frac{|z|}{\ell/2}\right]$$

$$E_\perp = \frac{-i\omega \sin\theta}{rc^2}\int \quad dz$$

$$\approx \frac{I_0 \ell}{2} e^{-i\omega t}\left(\frac{-i\omega \sin\theta}{rc^2}\right)$$

Whatever bus 

Poynting power (time avg) $\langle S \rangle = \frac{c}{4\pi}\left(\frac{I_0 \ell}{\lambda}\frac{\pi}{c}\right)^2 \frac{\sin^2\theta}{r^2} \langle e^{i\omega t^2} \rangle$ $\hookrightarrow \frac{1}{2}$

this avgs to 1/2 bc we really meant sin(ωt) the whole time
$\langle \sin^2(\omega t) \rangle = \frac{1}{2}$

$$P \propto \sin^2\theta$$

like the e^- in Larmor
bc we made assumptions $\ell \ll \lambda$, $e^{i\omega t}$
to make it so. But still Dipole!

Antennas #4

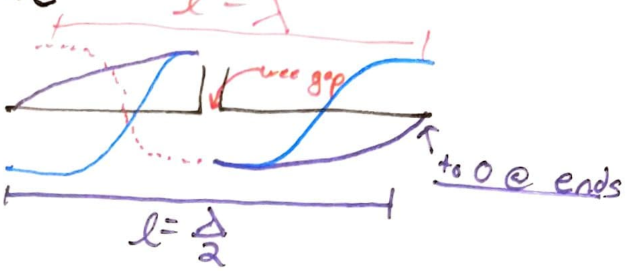
The total power. Int over θ sphere

$$\langle P \rangle = \frac{\pi^2}{3c}\left(\frac{I_0 l}{\lambda}\right)^2$$

length of dipole in wavelengths

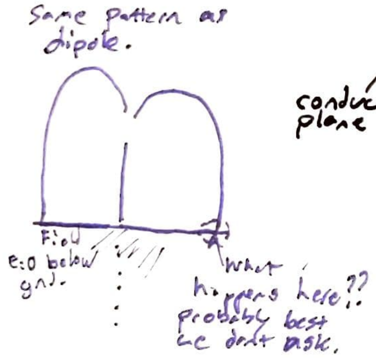
Now Dipoles @ Resonance

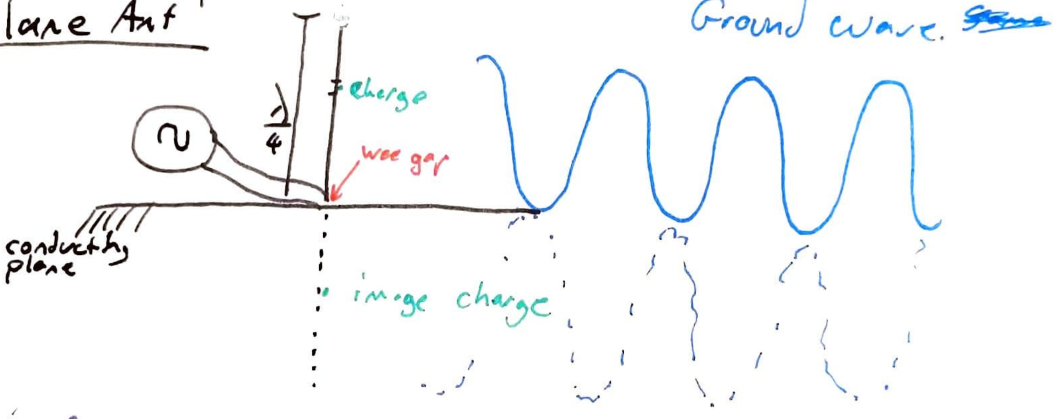
$$l \simeq \frac{\lambda}{2}$$

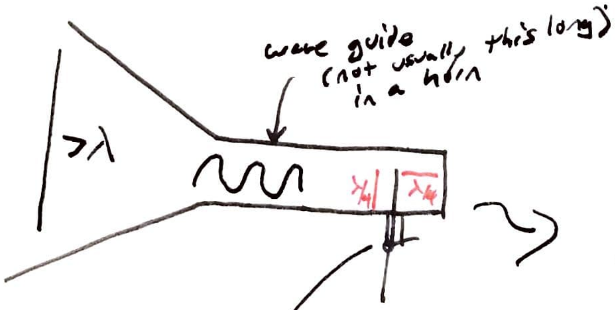


A magic dipole

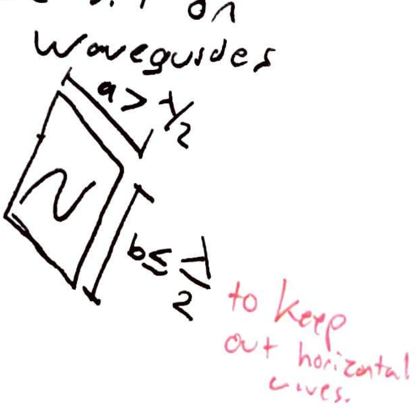
The Groundplane Ant







ERA see 3.4 on waveguides



Note Feed detail