RADIO ASTRONOMY
SES598
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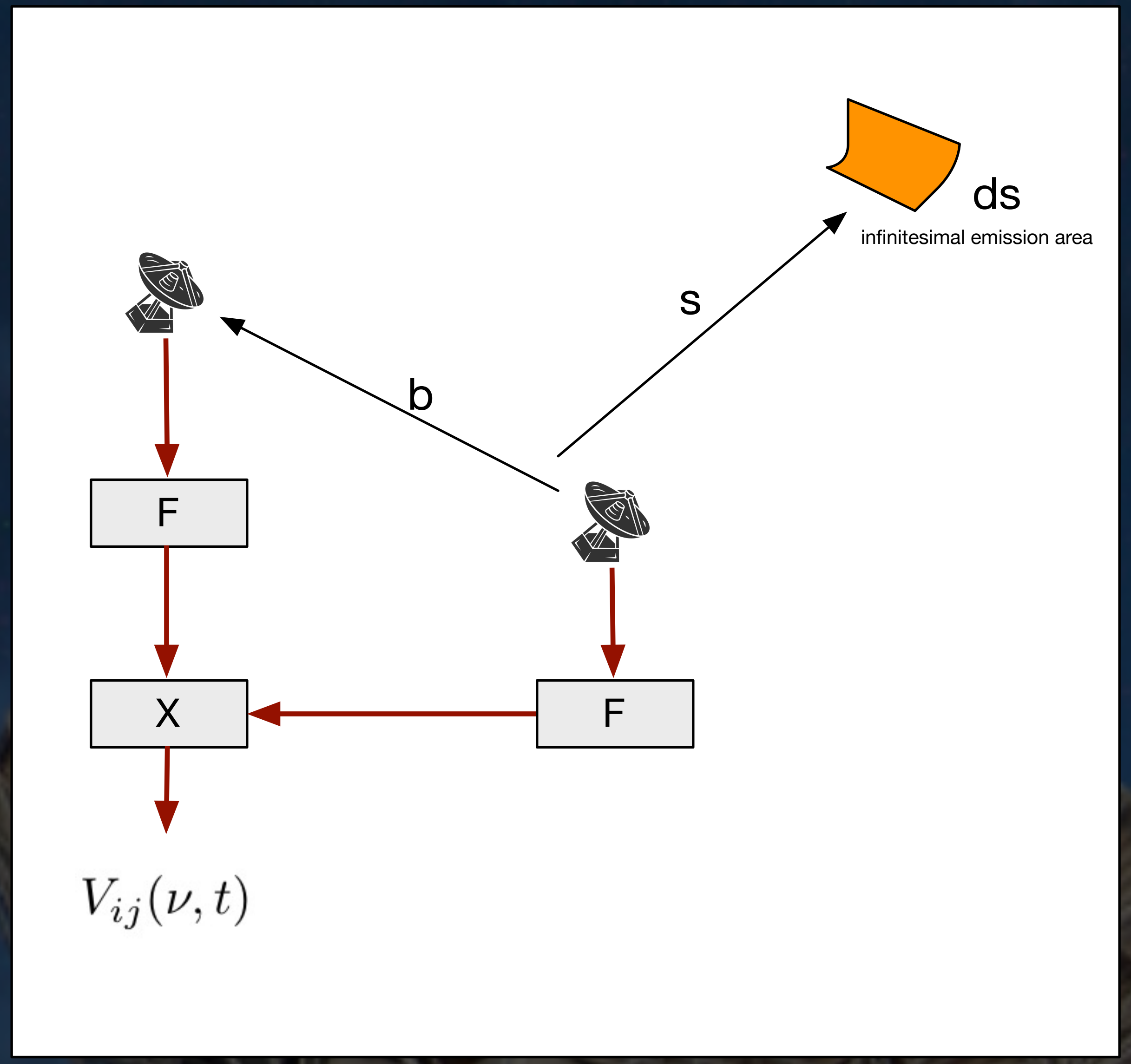
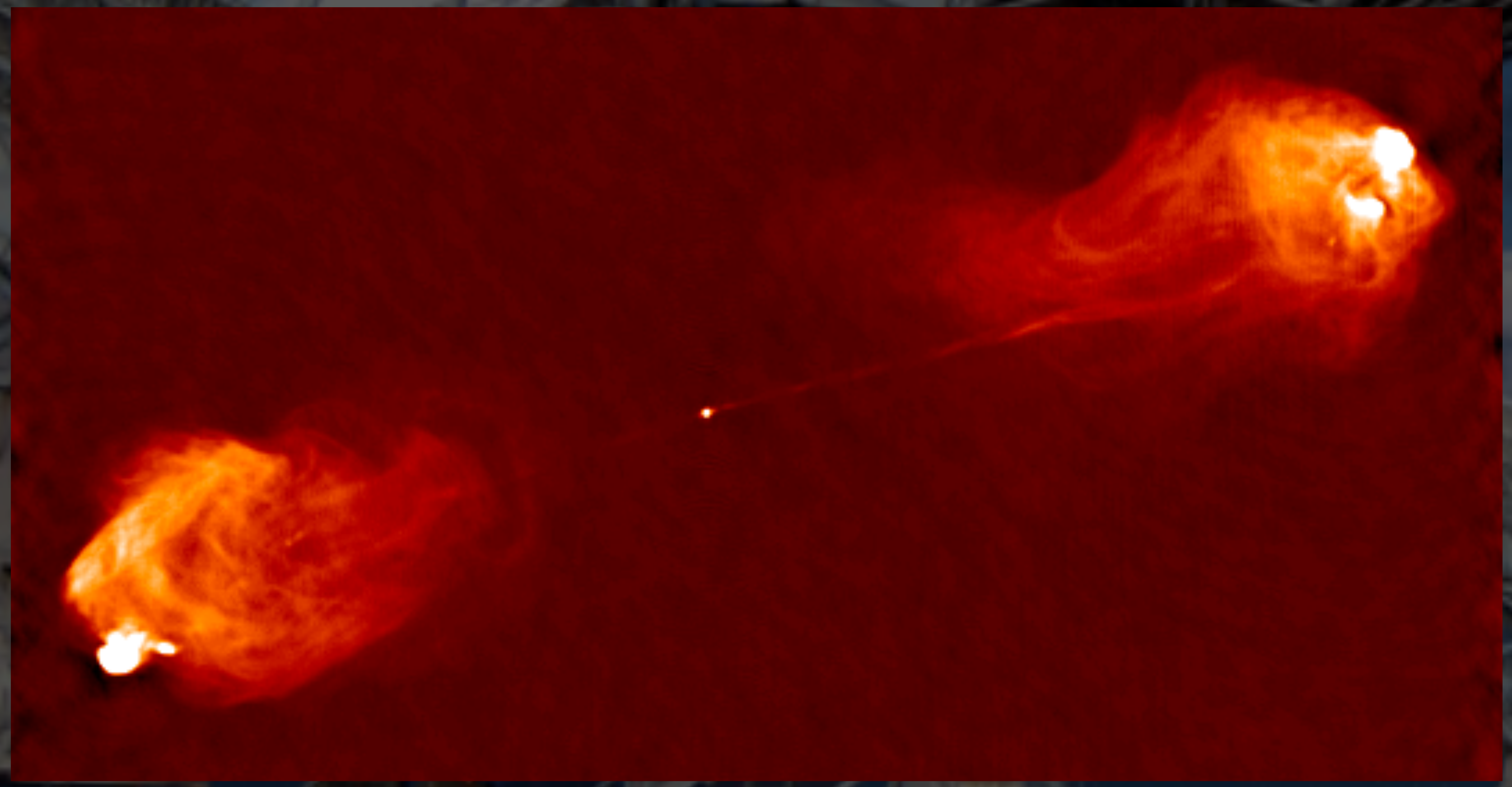
ARIZONA STATE UNIVERSITY

SYNTHESIS IMAGING

HOW DO WE GET FROM CROSS CORRELATIONS





TO AN IMAGE?

THE CROSS CORRELATION MEASURES THIS

$$V_{ij}(\nu, t) = \int_{4\pi} g_i g_j A_i(\hat{s}, \nu) I(\hat{s}, \nu) A_j(\hat{s}, \nu) e^{-i\pi \vec{b}_{ij} \cdot \hat{s} \frac{\nu}{c}} ds^2$$

Instrument Gains

Direction dependent gain

Flux on the sky

Baseline vector

Direction on sky

Infinitesimal sky area

SIMPLIFY

Assume we can solve for gains somehow $g \to 1$

Assume our object is near flat part of beam $A \to 1$

Let b/lambda = (u,v,w) and s = (l,m)*

$$V_{ij}(\nu, t) \approx \int_{4\pi} I(\hat{s}, \nu) e^{-i\pi(u_{ij}l + v_{ij}m + w_{ij}\sqrt{1-l^2-m^2})} \frac{dldm}{\sqrt{1 - l^2 - m^2}}$$

*recall $\hat{s}$ is a unit vector

$$V_{ij}(\nu, t) \approx \int_{4\pi} I(\hat{s}, \nu) e^{-i\pi(u_{ij}l + v_{ij}m + w_{ij}\sqrt{1-l^2-m^2})} \frac{dldm}{\sqrt{1 - l^2 - m^2}}$$

If w is small and l,m are small

$$V_{ij}(\nu, t) = \int_{4\pi} I(\hat{s}, \nu) e^{-i\pi(u_{ij}l + v_{ij}m)} dldm$$

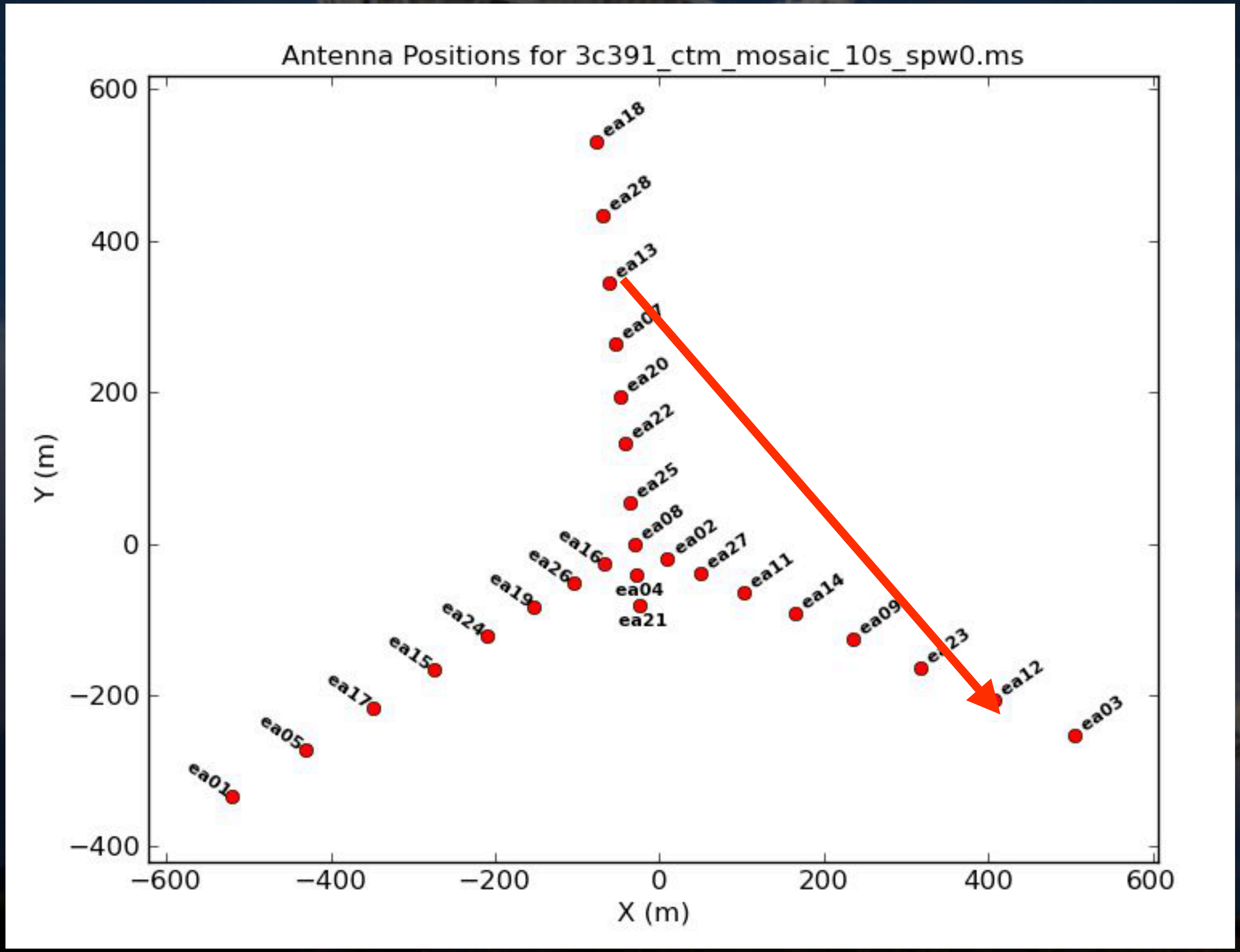
The Fourier Transform!

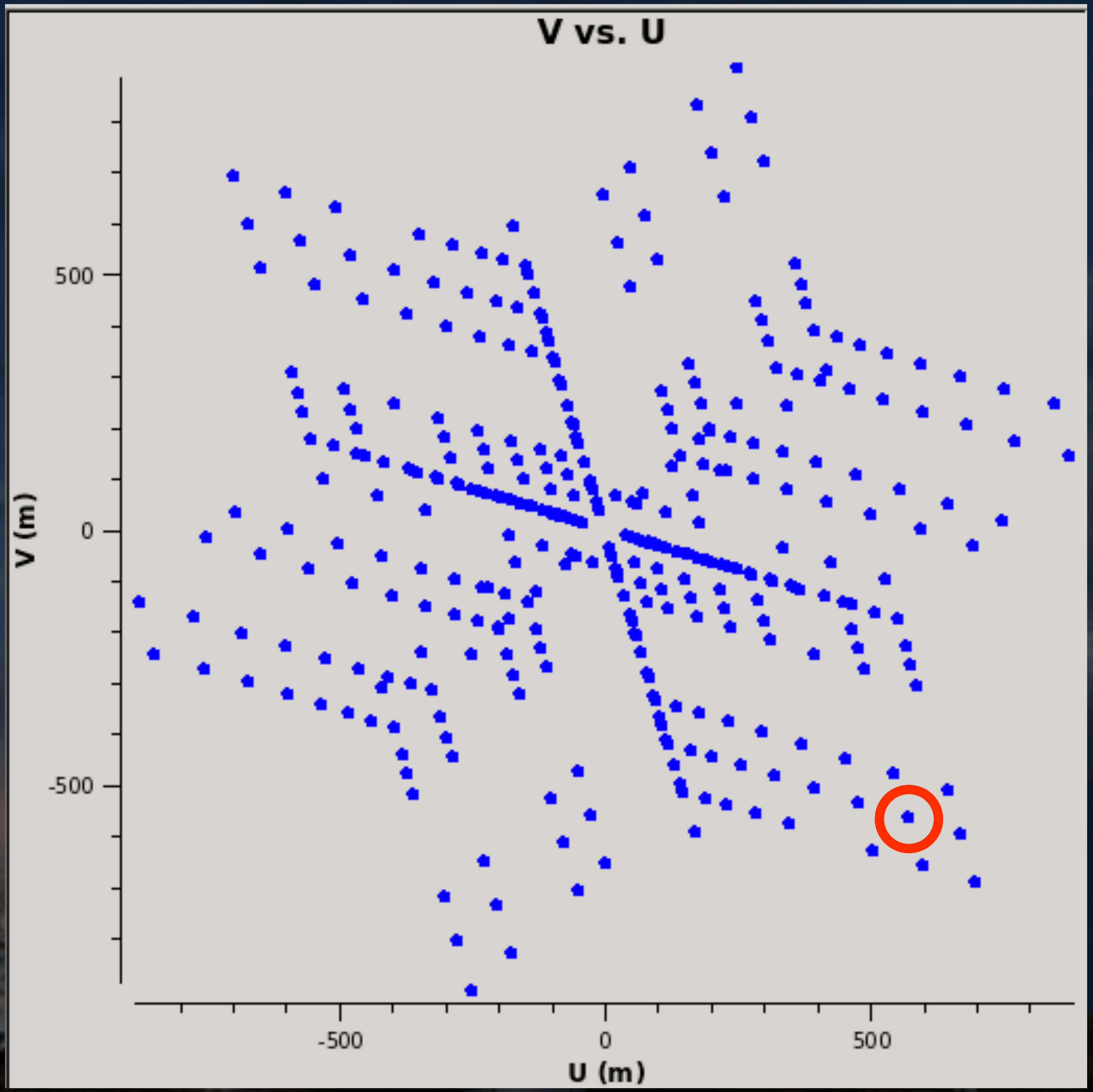
In the convolution theorem

$$V_{ij} = \tilde{I}((u, v)) \cdot \delta(u - u_{ij}, v - v_{ij})$$

the FT of the true sky times deltas at the baselines

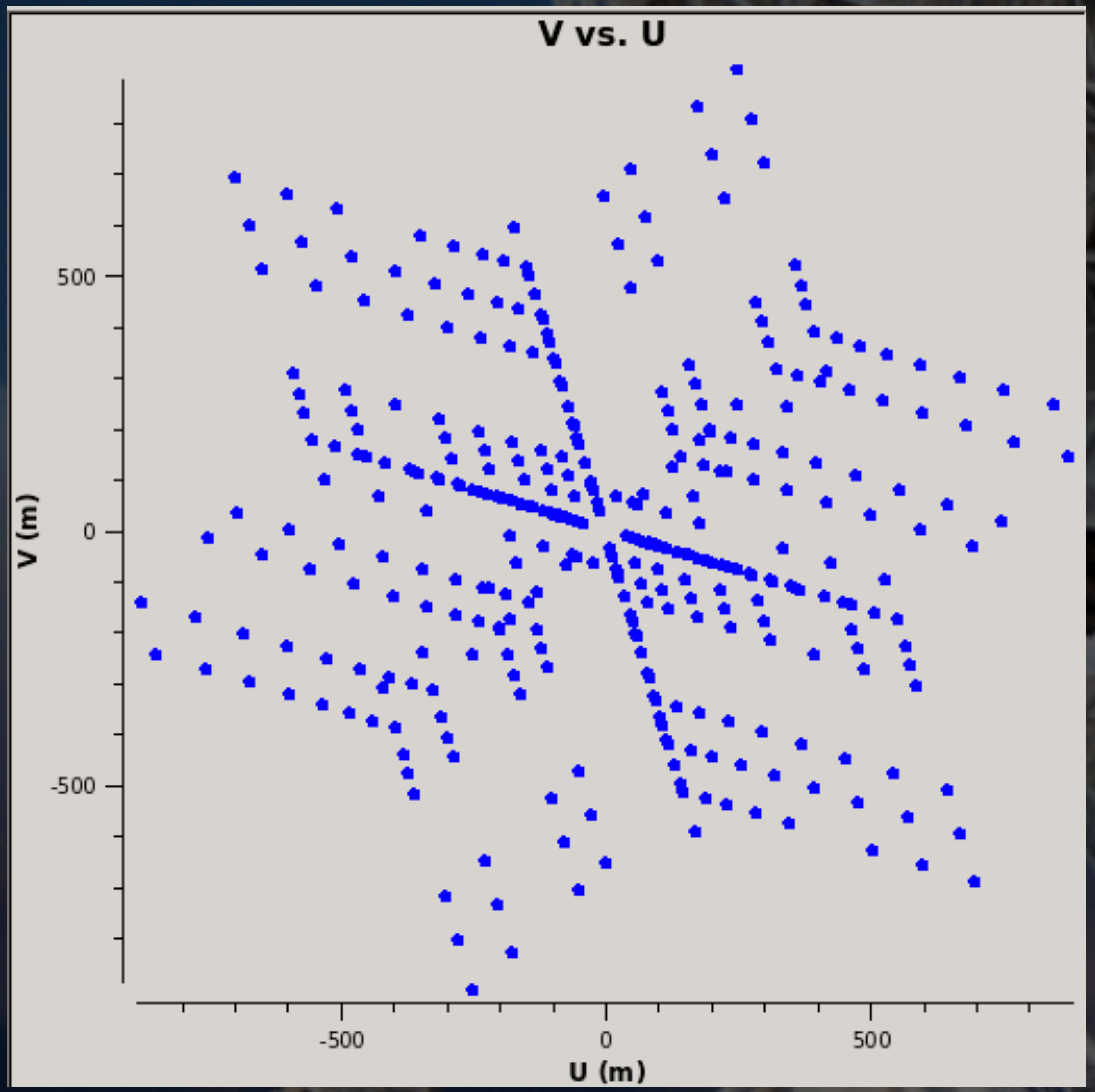
EACH BASELINE MAPS TO A MODE IN UV SPACE



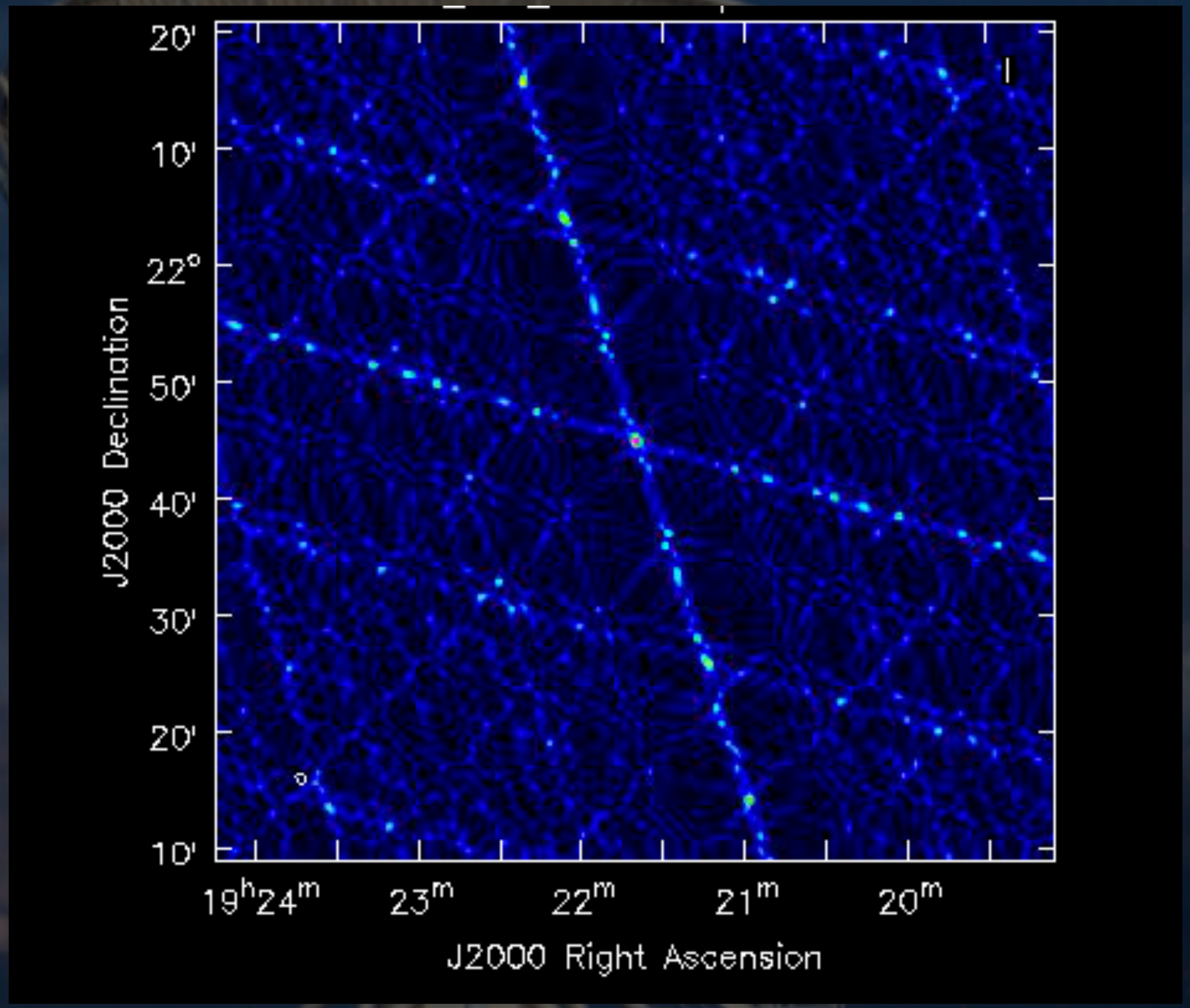


Suppose we a 1Jy src at $\hat{s} \cdot \vec{b} = 1$

$$V_{ij} = 1$$



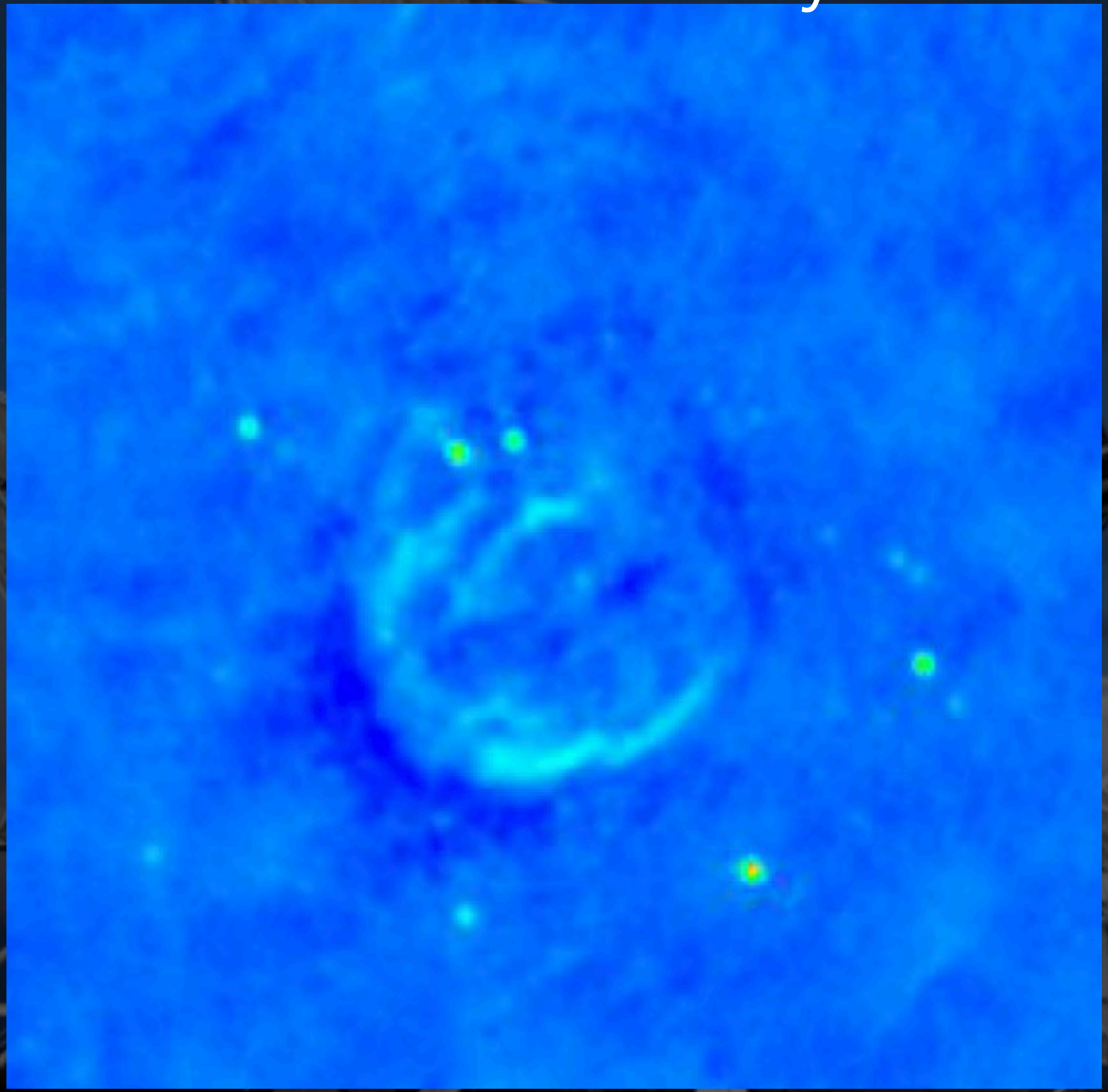
1. Grid V_{ij}
2. Fourier Tranform



The Point Spread Function

The interferometer measures _this_

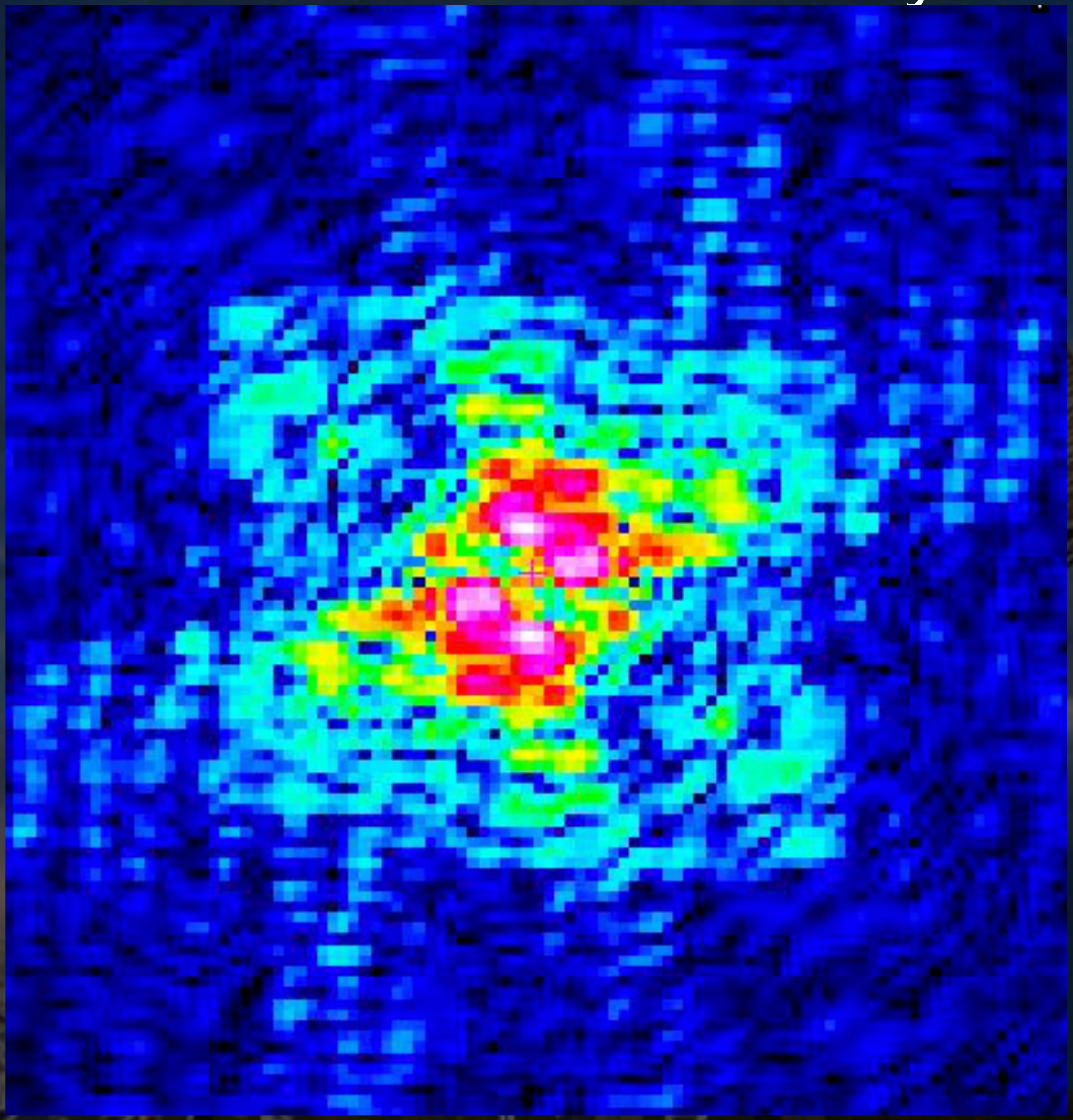
The “True” sky



$I(\hat{s}, \nu)$

$\xleftarrow{F^{-1}}$

The “True” Fourier sky



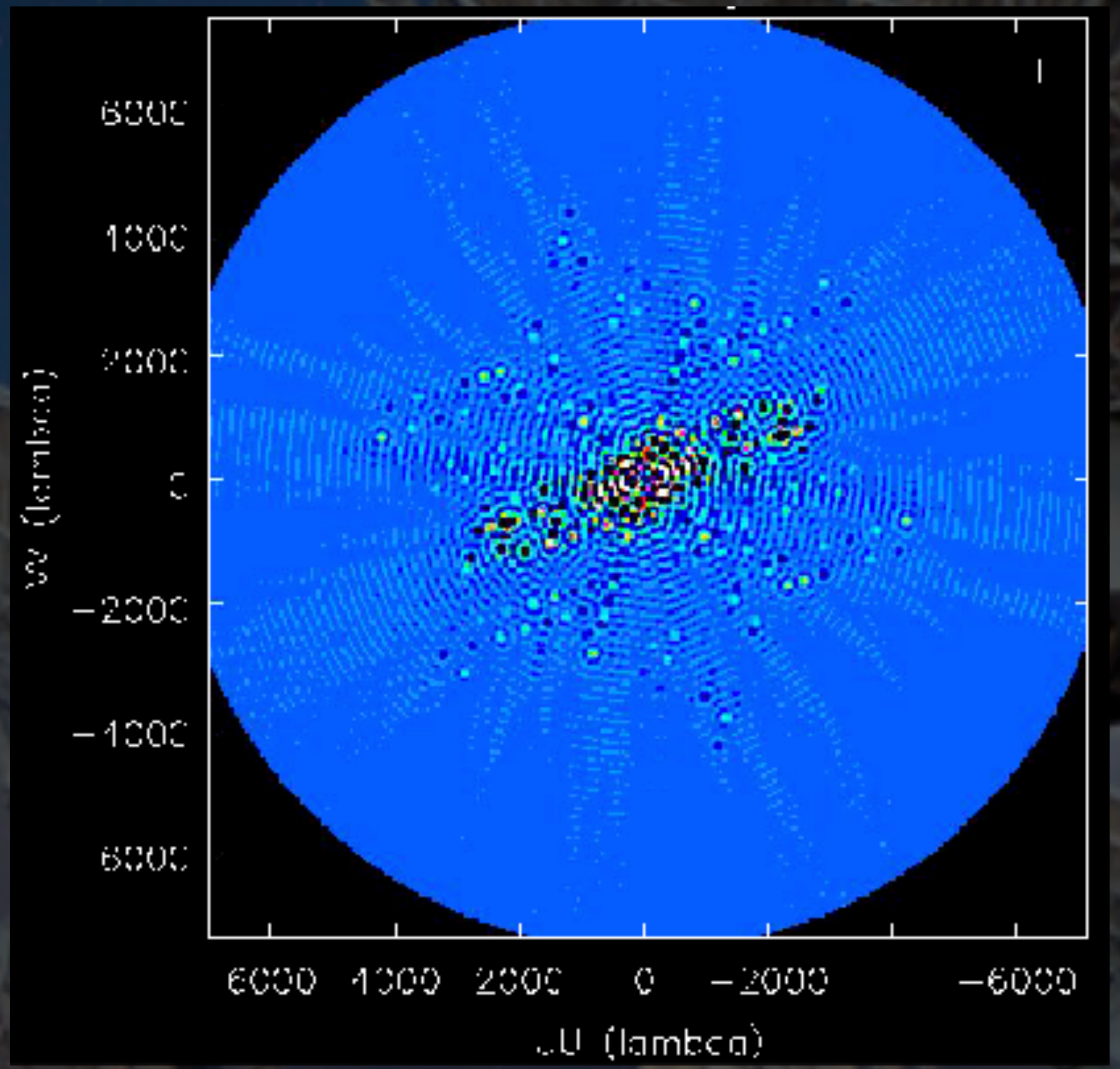
$\tilde{I}((u, v), \nu)$

F(sky * fringe modes)

$$V_{ij}(\nu, t) = \int I(\hat{s}, \nu) e^{-i\pi(u_{ij} l + v_{ij} m)} dl dm$$

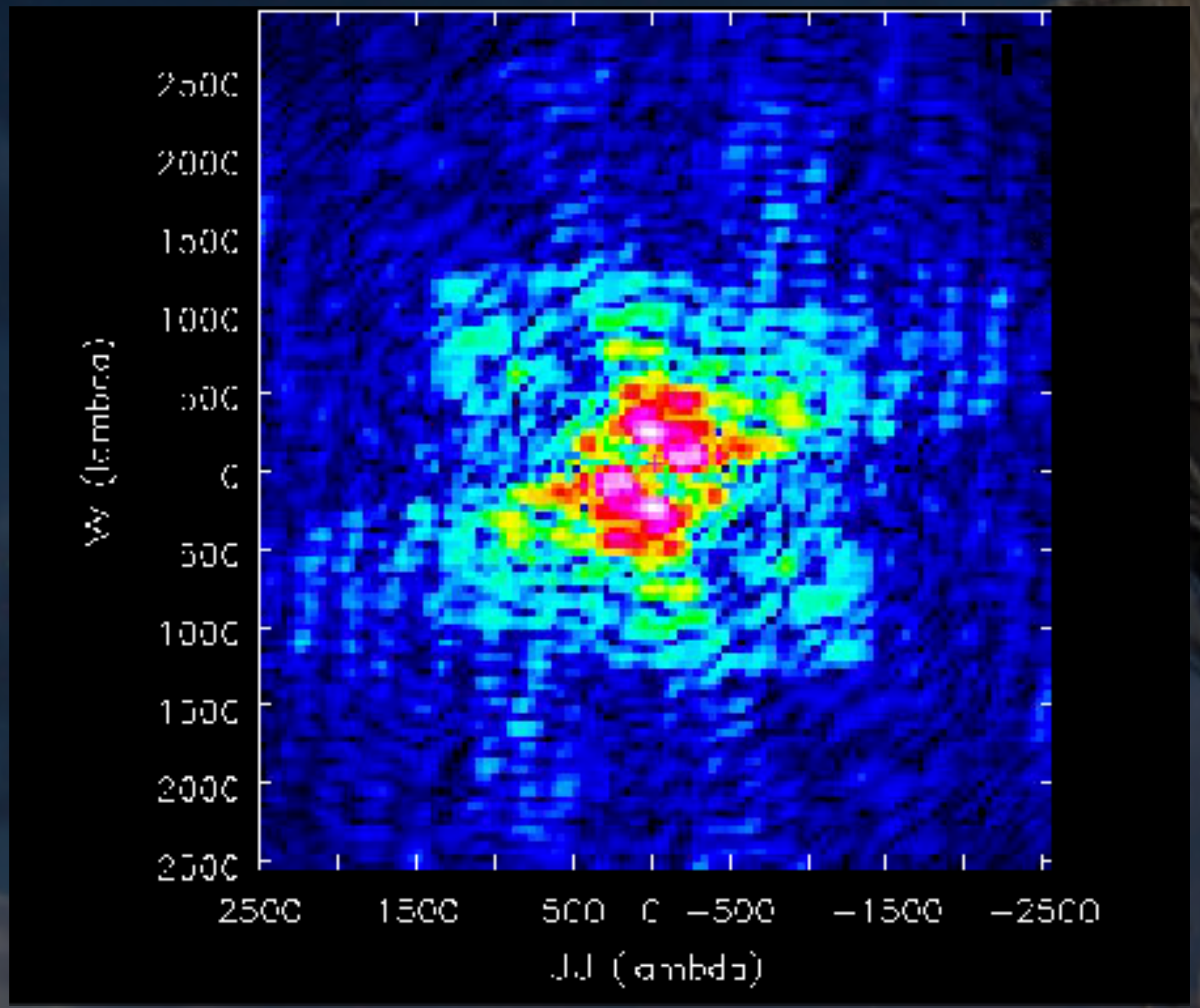
F^-1(sky) * Fringe deltas

$$V_{ij} = \tilde{I}((u, v)) \cdot \delta(u - u_{ij}, v - v_{ij})$$



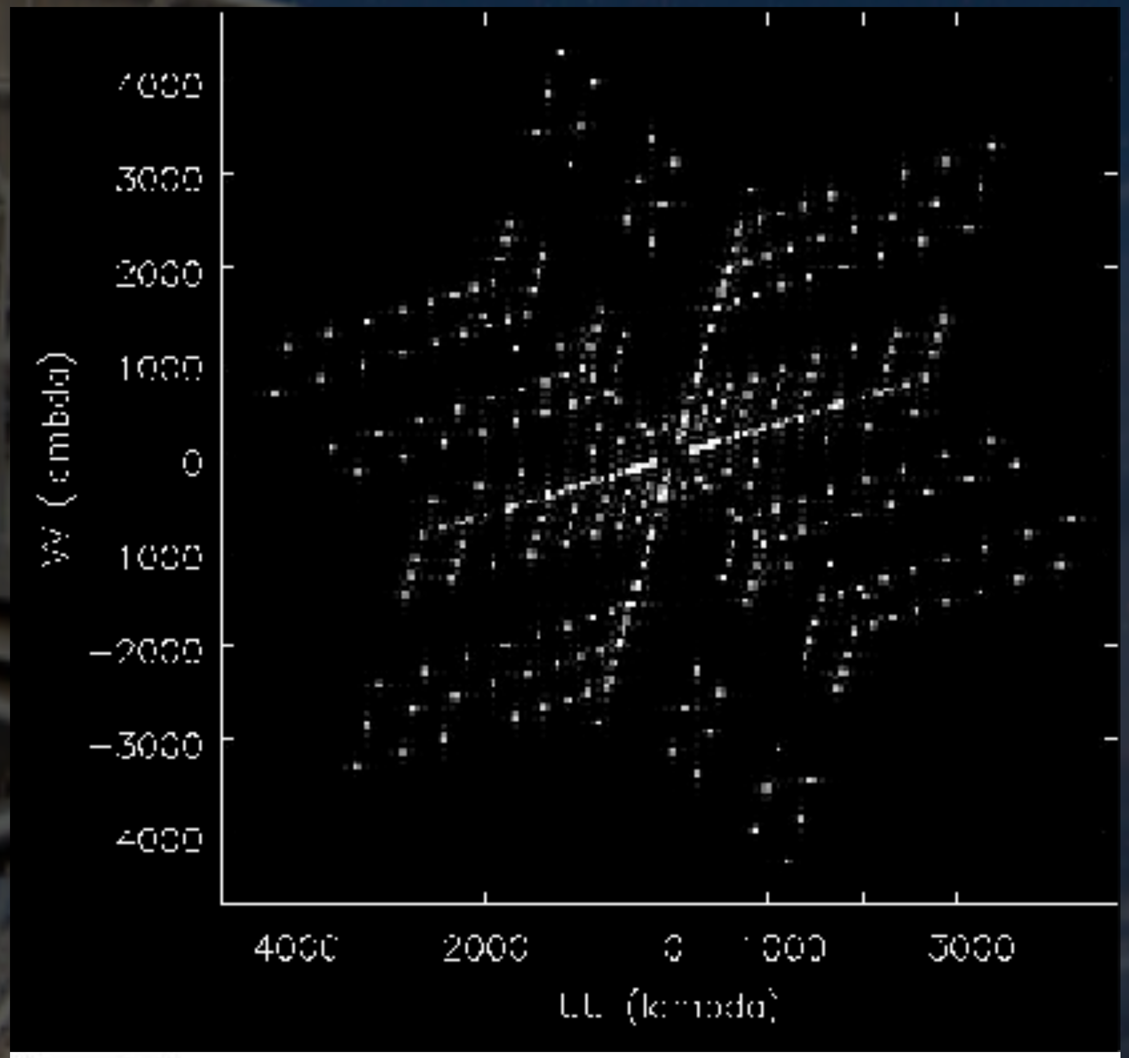
$$V(u_{ij}, v_{ij})$$

=



$$\tilde{I}((u, v))$$

X



$$\sum_{ij} \delta(u - u_{ij}, v - v_{ij})$$

The "True" sky
The "True" Fourier sky
F
X aperture function
Measured Sky
Traditionally known as the *dirty image*
F^{-1}

CLEAN = USE WHAT WE KNOW TO MAKE AN INFORMED GUESS ABOUT THE TRUE SKY

IE solve the inverse problem. But Which one!?

Our rough model

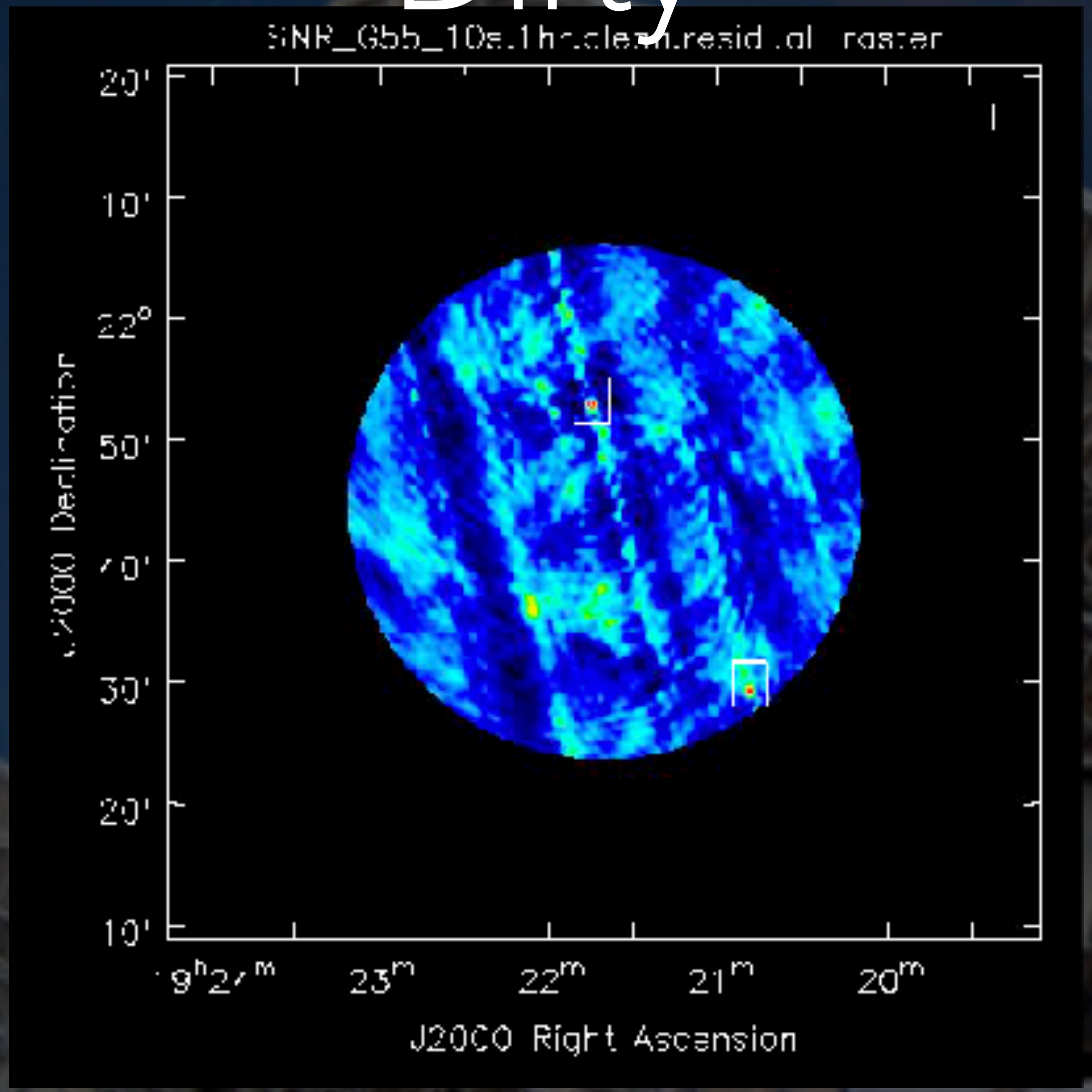
$$V_{ij}(\nu, t) = \int_{4\pi} I(\hat{s}, \nu) e^{-i\pi(u_{ij}l + v_{ij}m)} dldm$$

A slightly more accurate one

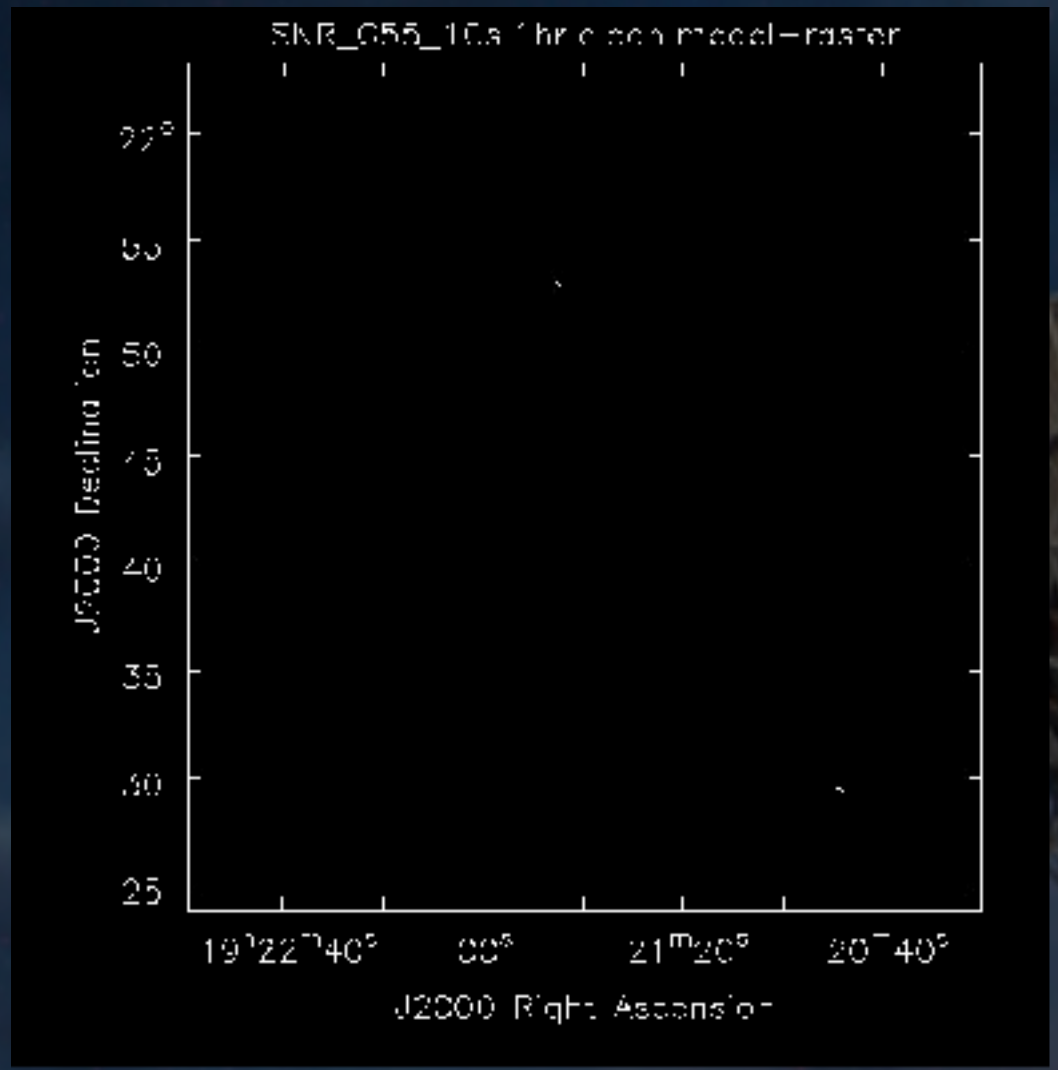
$$V_{ij}(\nu, t) \approx \int_{4\pi} I(\hat{s}, \nu) e^{-i\pi(u_{ij}l + v_{ij}m + w_{ij}\sqrt{1-l^2-m^2})} \frac{dldm}{\sqrt{1 - l^2 - m^2}}$$

CLEAN

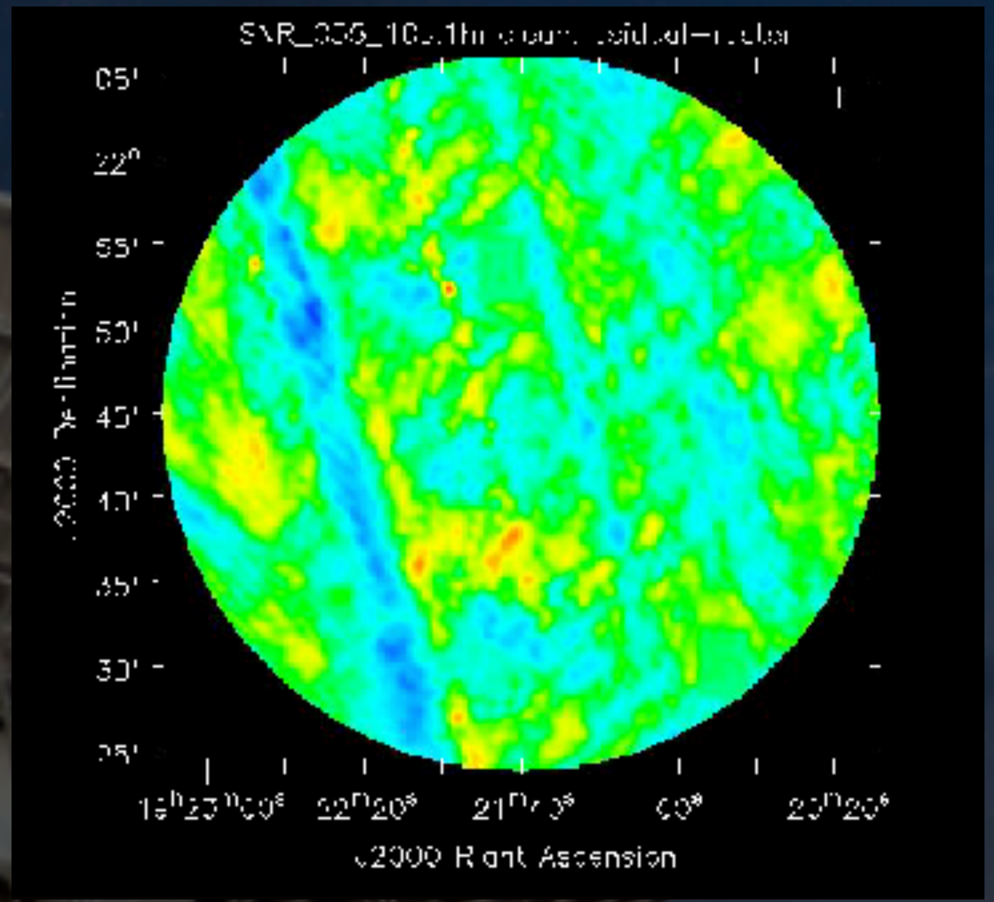
Dirty



Model



Residual



$$V_{ij}(\nu,t) = \int_{4\pi} I(\hat{s},\nu)e^{-i\pi(u_{ij}l+v_{ij}m)}dldm$$

or

$$V_{ij}(\nu,t) \approx \int_{4\pi} I(\hat{s},\nu)e^{-i\pi(u_{ij}l+v_{ij}m+w_{ij}\sqrt{1-l^2-m^2})}\frac{dldm}{\sqrt{1-l^2-m^2}}$$

Identify most likely point sources

1. By human prior
2. By heuristic, like "brightest pixel"

Add fraction of that flux to model.

Forward model visibilities and subtract from data

Image the difference (aka the "residuals")

Repeat

TYPES OF CLEAN

Högbom

$$V_{ij}(\nu,t) = \int_{4\pi} I(\hat{s},\nu)e^{-i\pi(u_{ij}l+v_{ij}m)}dldm$$

Entirely in image plane

Assumes PSF is constant across image

i.e. w=0, flat sky

Cotton-schwab

$$V_{ij}(\nu,t) \approx \int_{4\pi} I(\hat{s},\nu)e^{-i\pi(u_{ij}l+v_{ij}m+w_{ij}\sqrt{1-l^2-m^2})}\frac{dldm}{\sqrt{1-l^2-m^2}}$$

Runs full forward model

Slow as molasses (relatively speaking, mostly fine these days)

The default is usually to combine the two

COTTON-SCHWAB

HÖGBOM

MINOR CYCLE

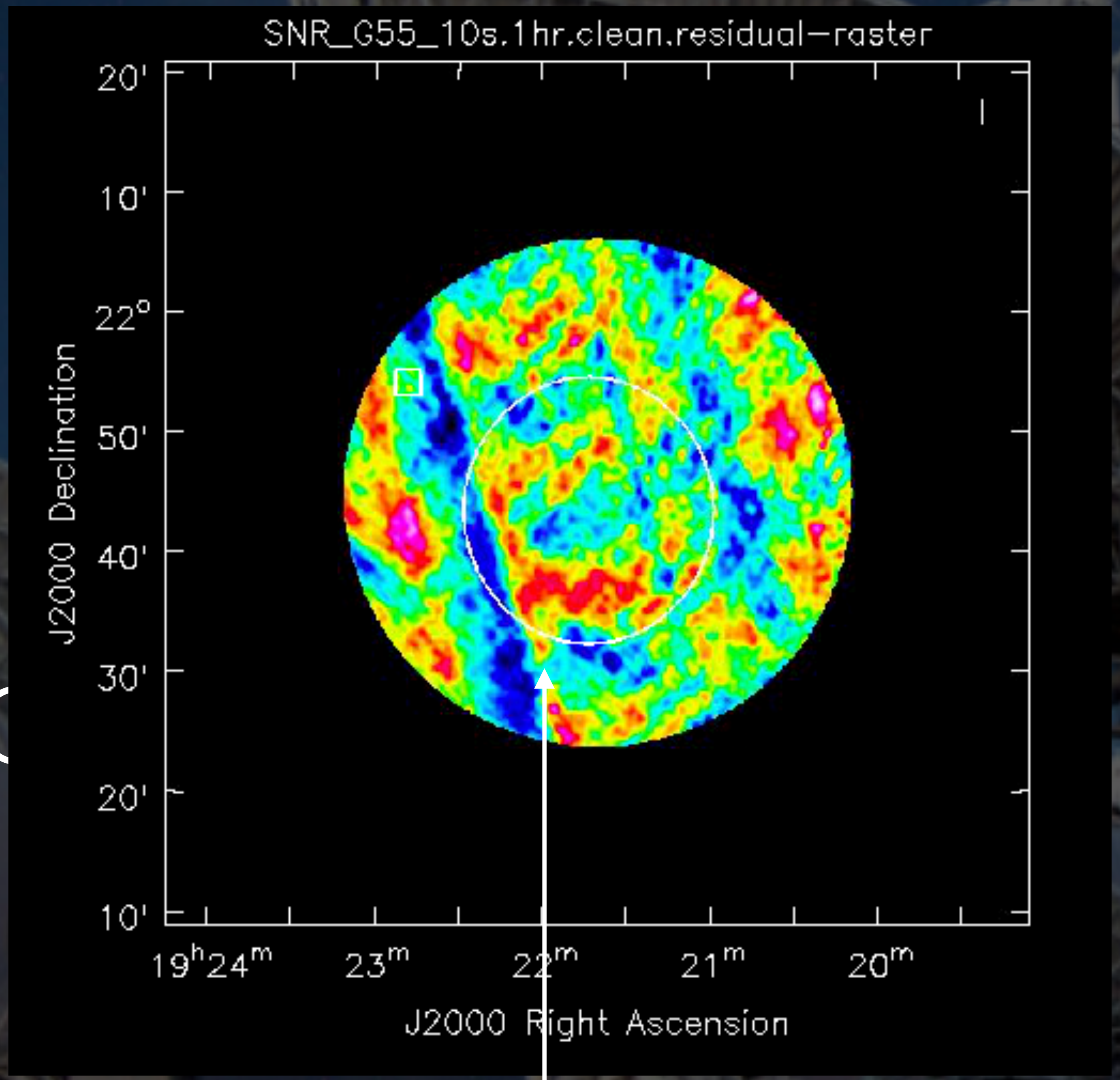
MAJOR CYCLE

Other variants:

- Bayesian reconstruction
- Maximum Entropy
- w projection
- Multiscale
- aw projection

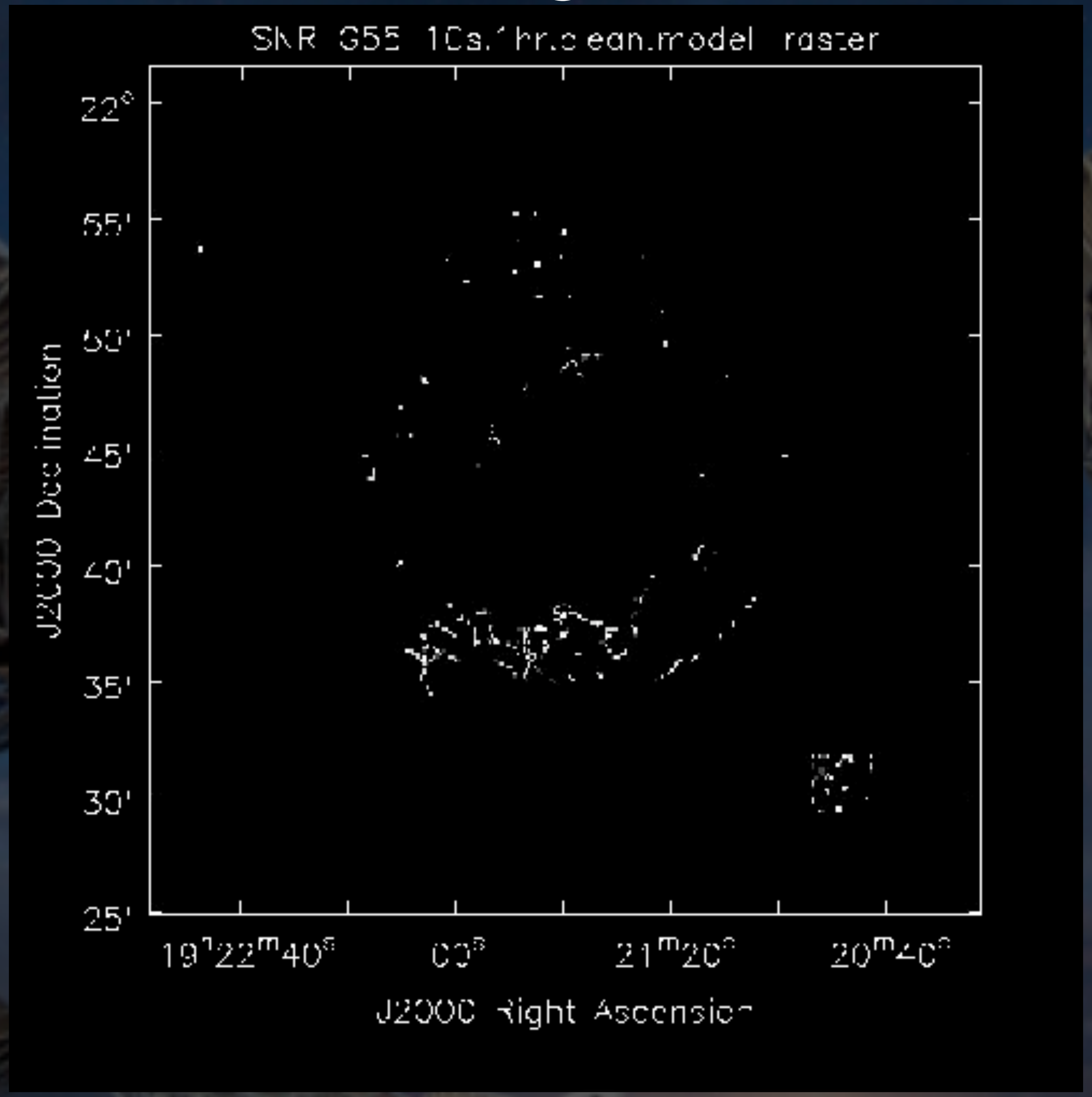
CLEAN CAN BE ABUSED!

More clean



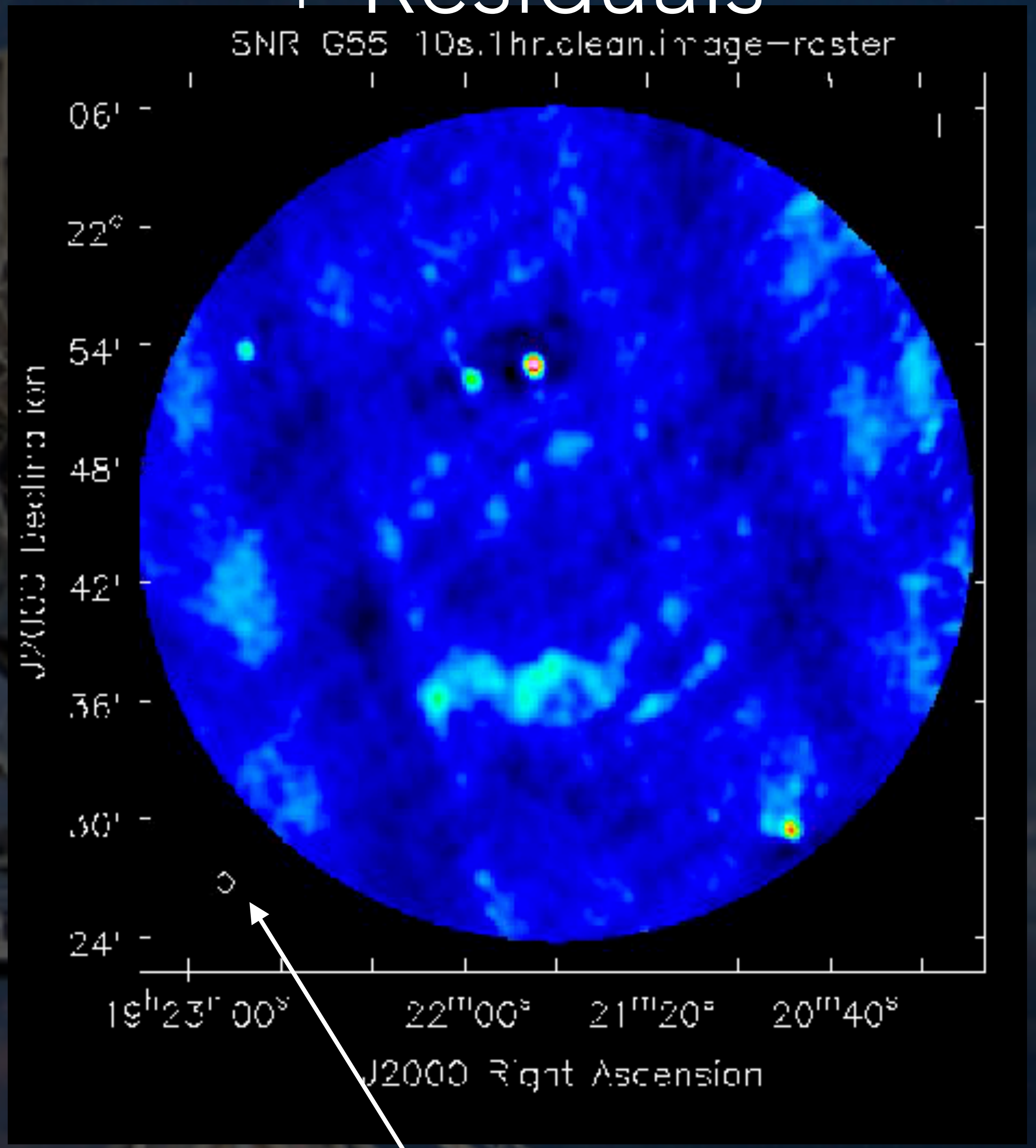
Guess that the object is in the inner half of the beam

Resulting Model



each pixel is a "clean model component"

Gaussian PSF · Model + Residuals

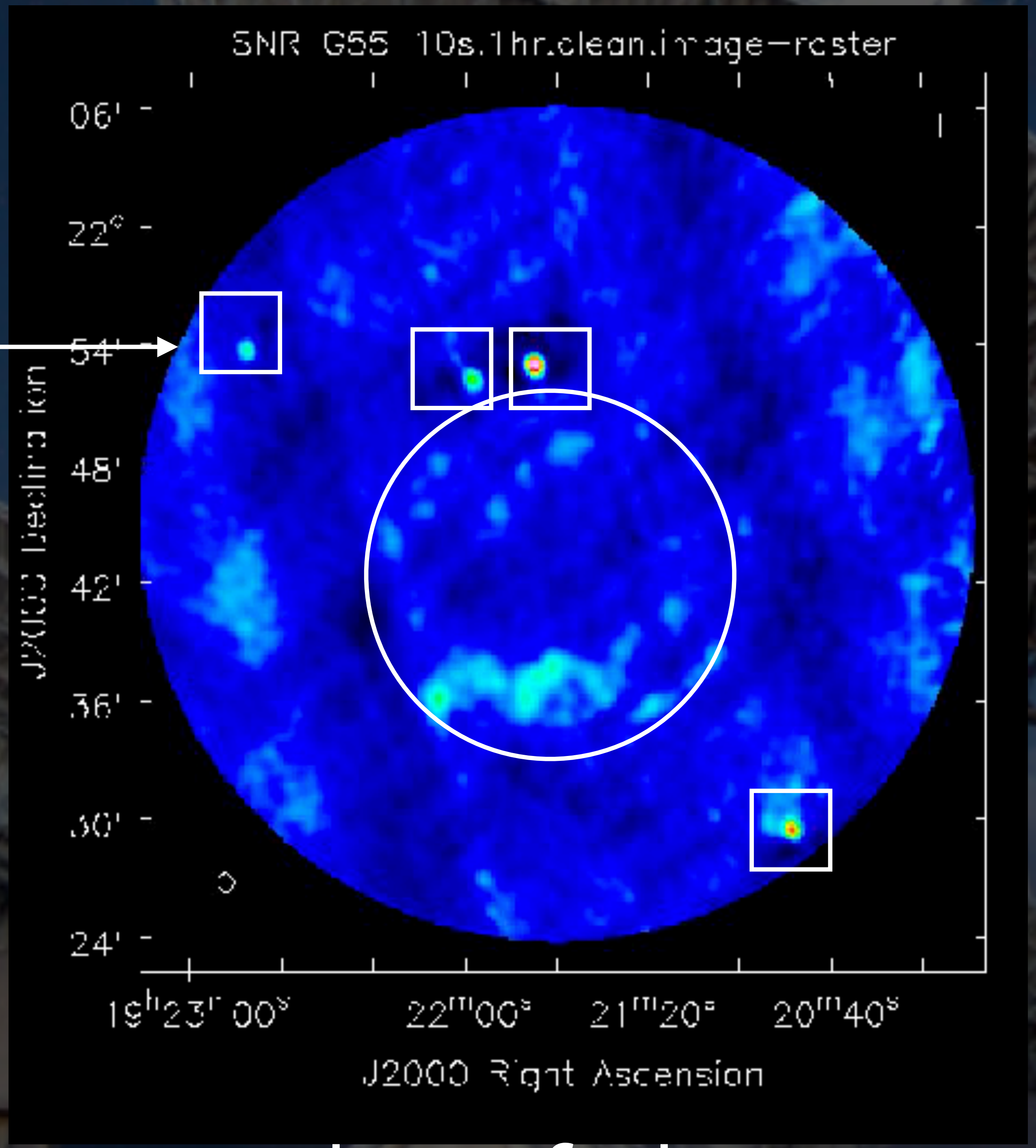


Gaussian fit to PSF aka "restoring beam"

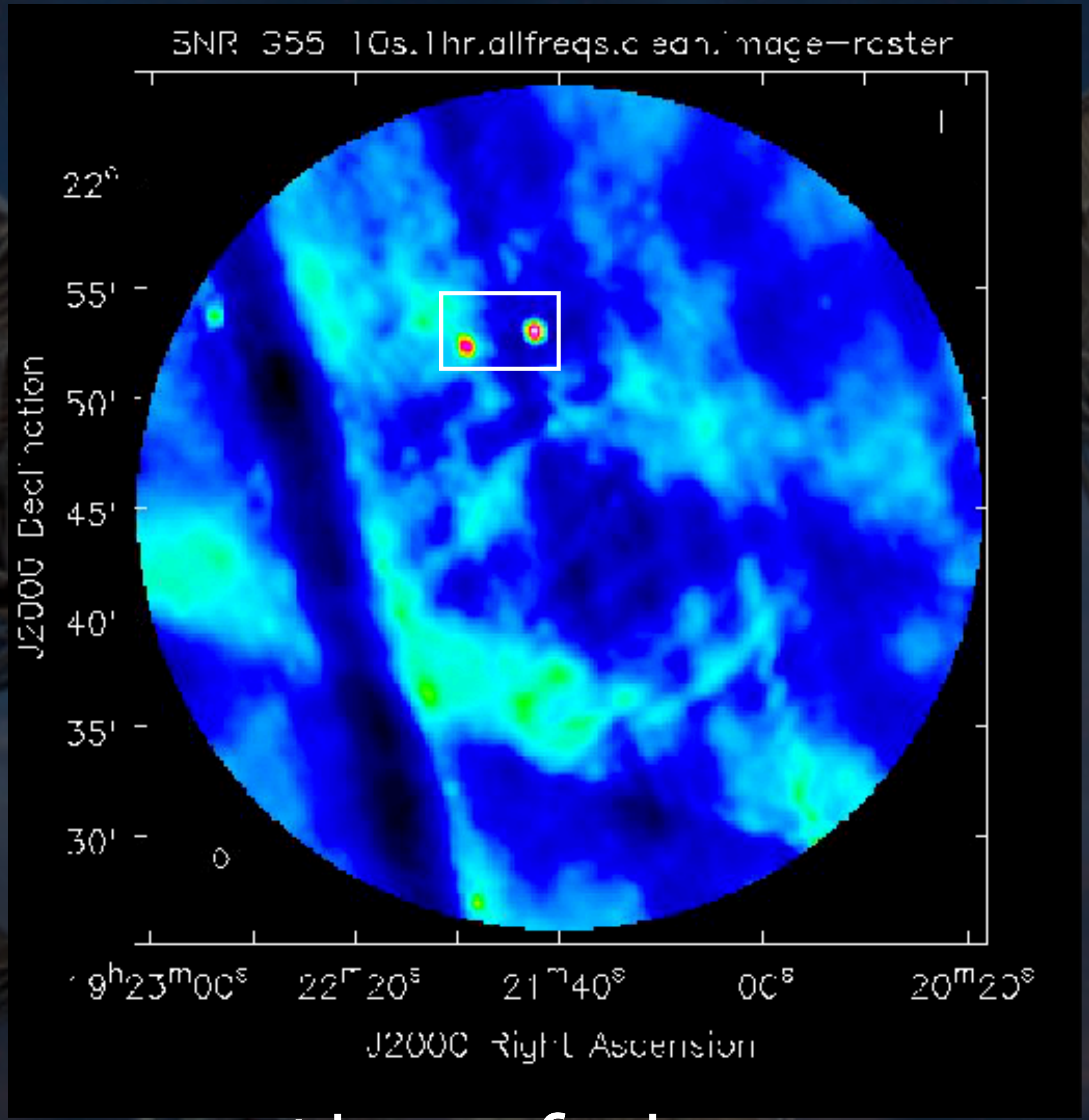
CLEAN USES YOUR PRIORS

CLEAN

Boxes



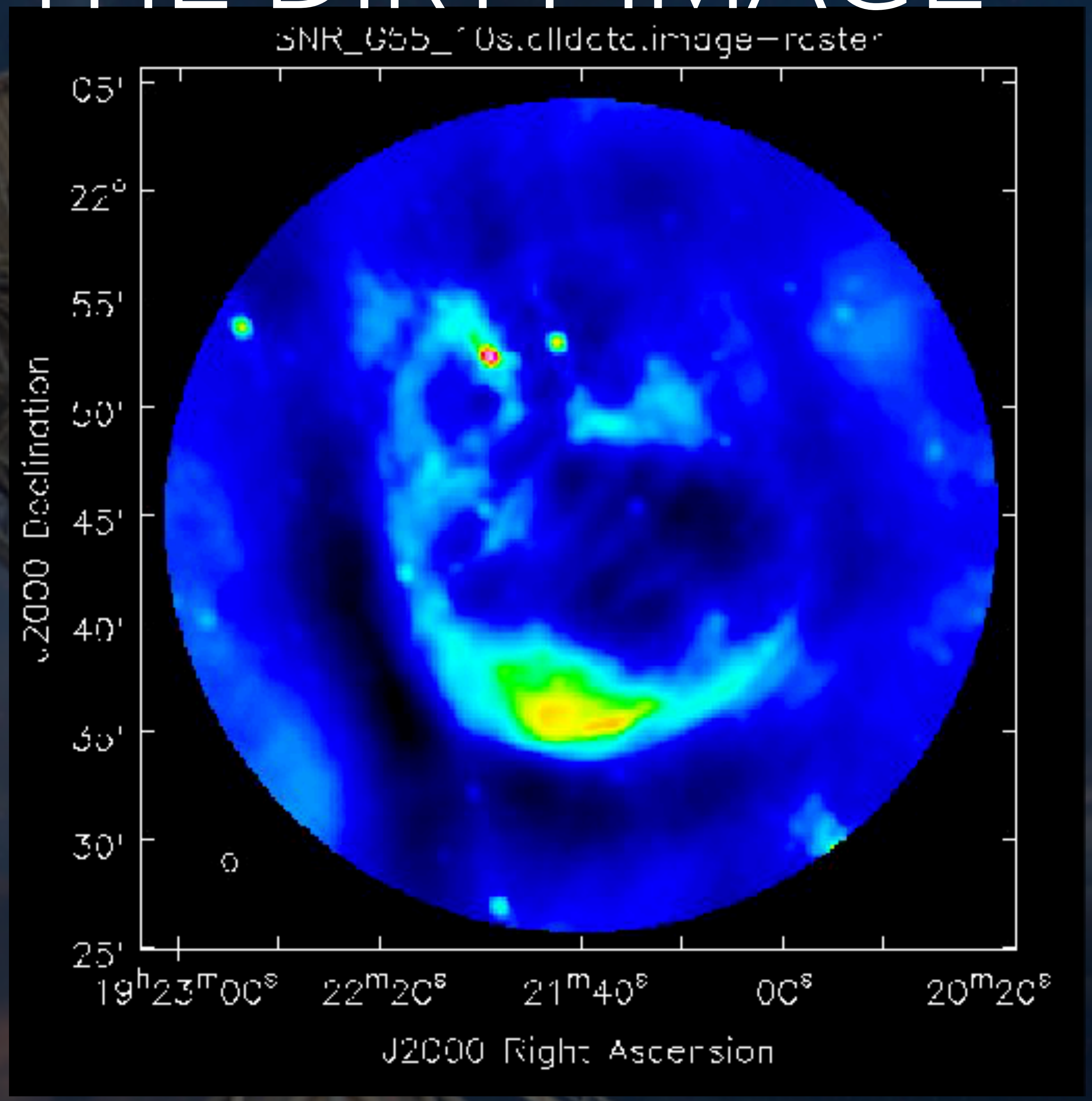
1hr of data



1hr of data

+ all frequencies

THE DIRTY IMAGE

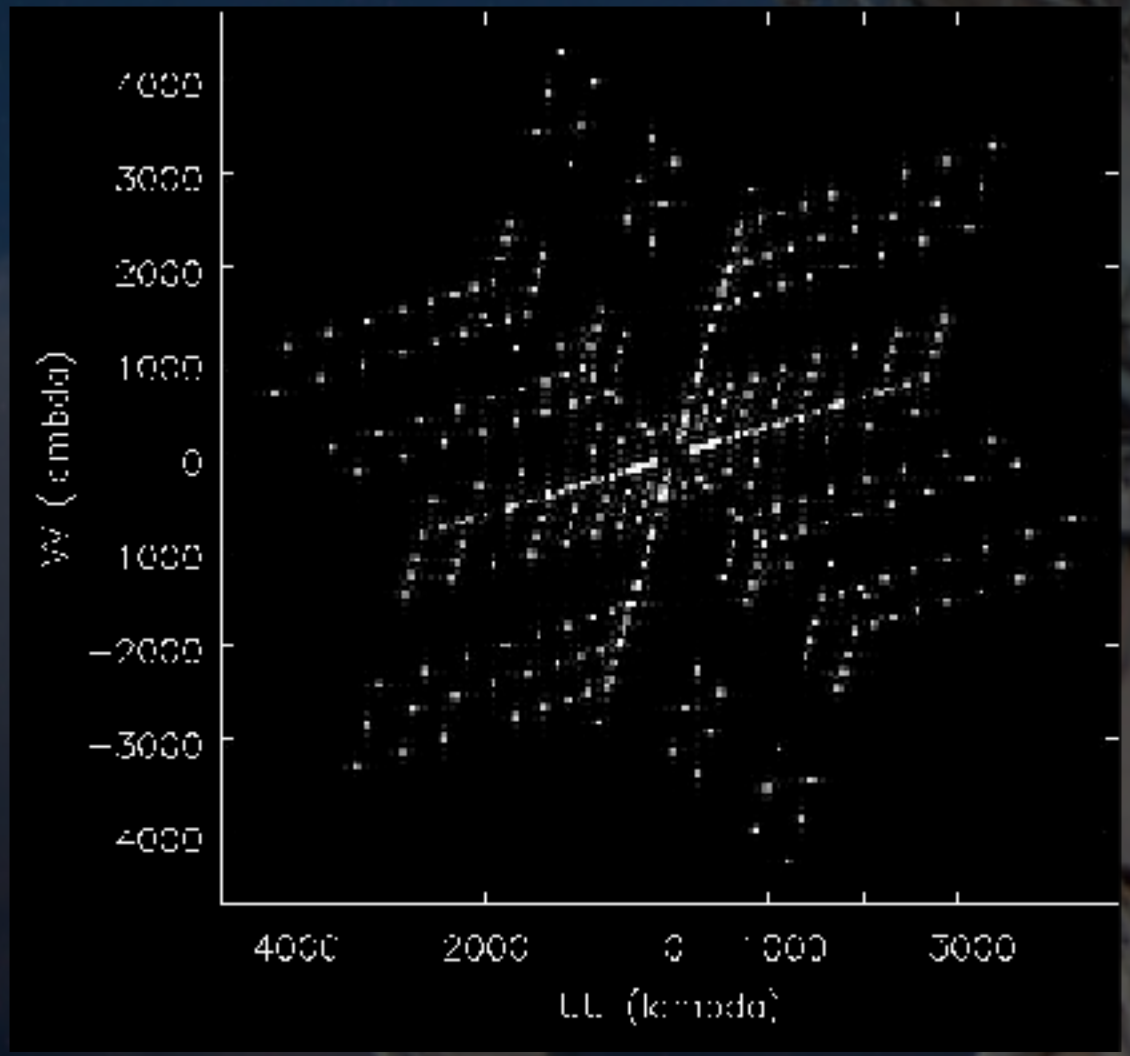


9hrs of data

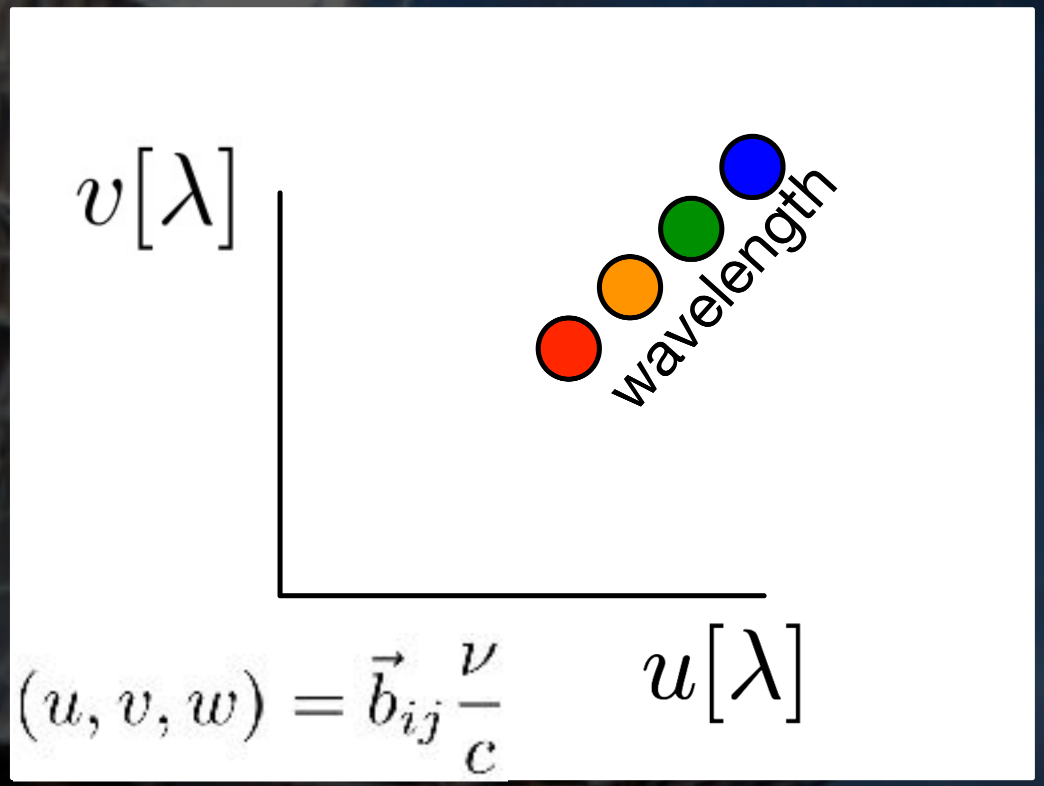
+ all frequencies

MORE MODES = MORE MEASUREMENTS *WITH SOME BIG ASSUMPTIONS

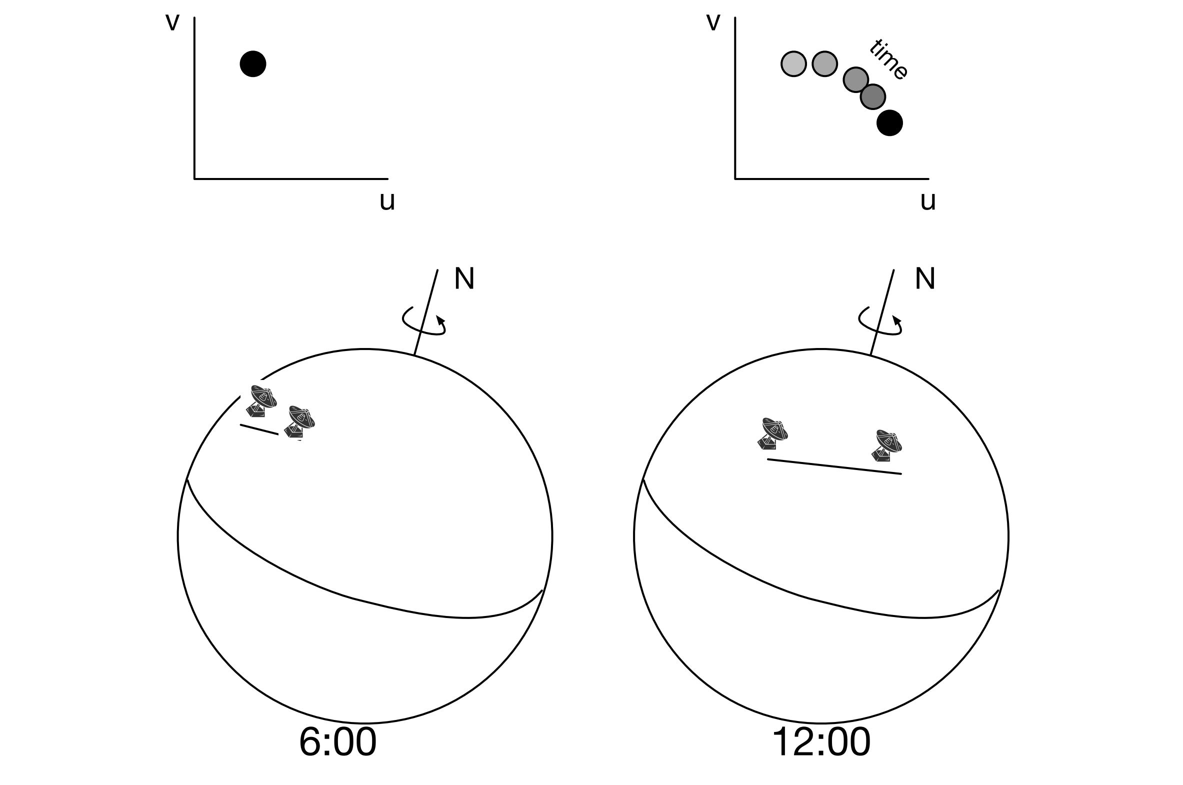
one integration



Assuming *constant in frequency*

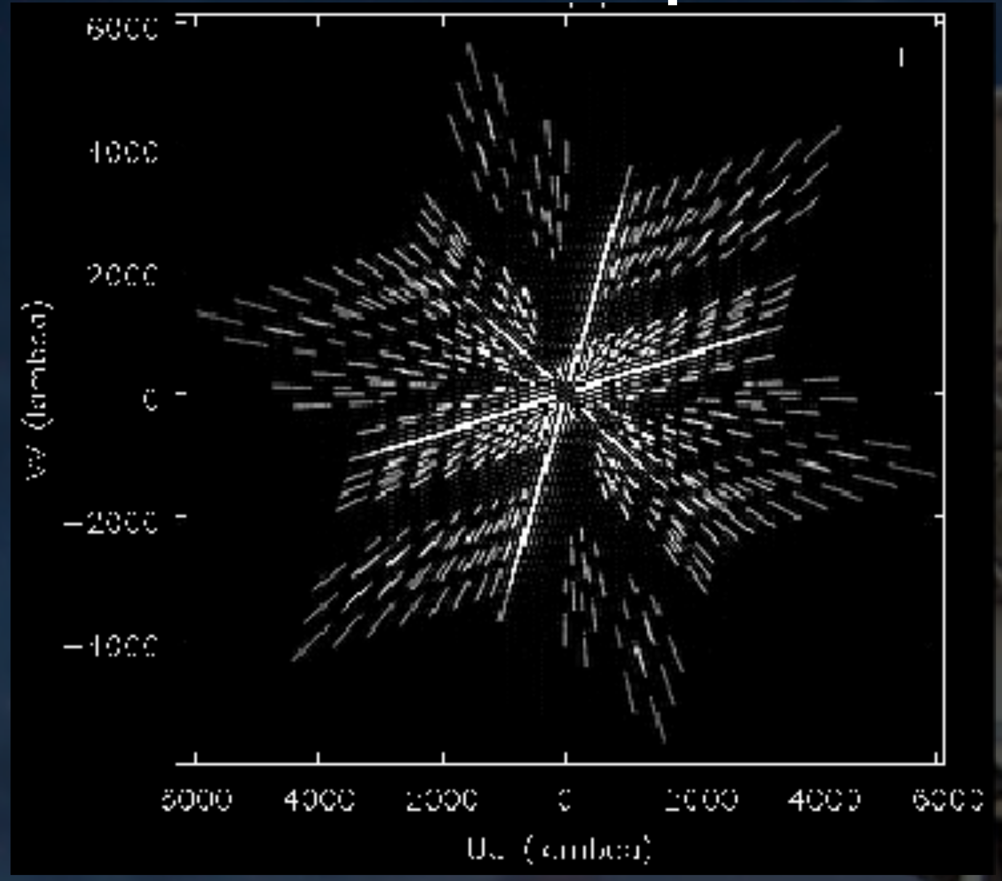


Assuming *constant in time*

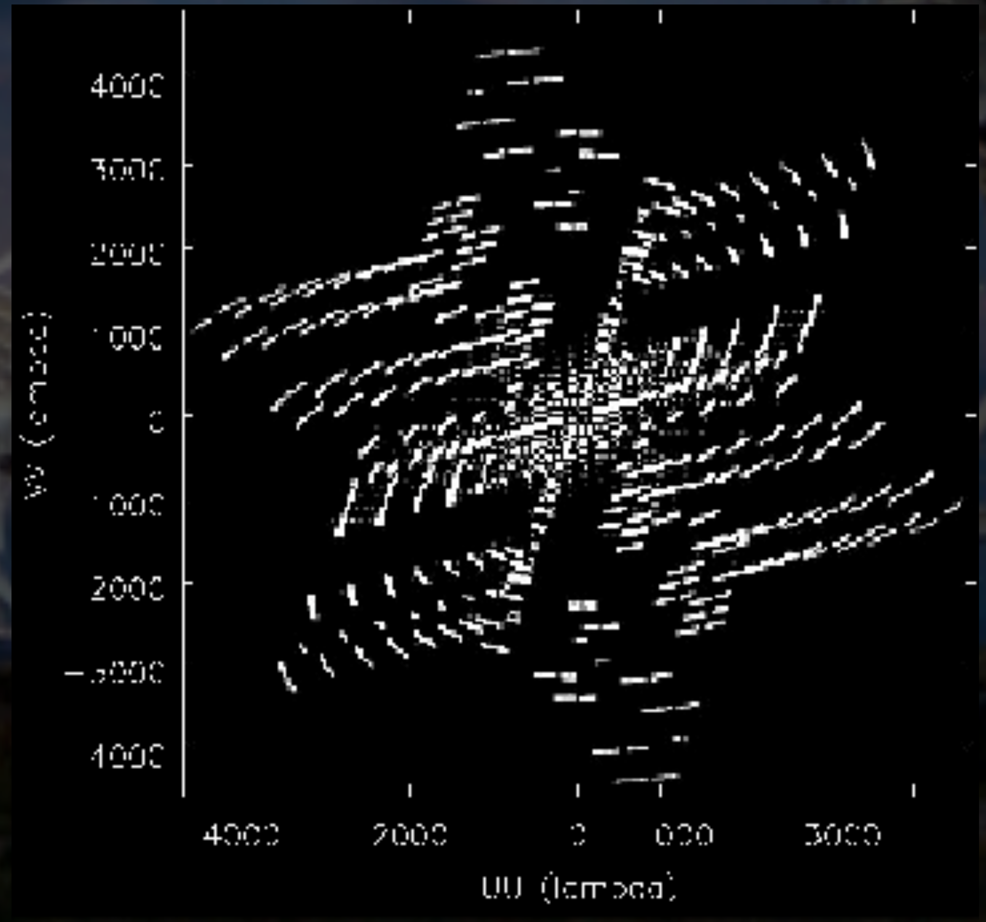


Baseline rotation relative to the fixed stars

Multi-frequency



Earth rotation



All combined