RA Lecture 3, D. Jacobs 11 Jan 2021

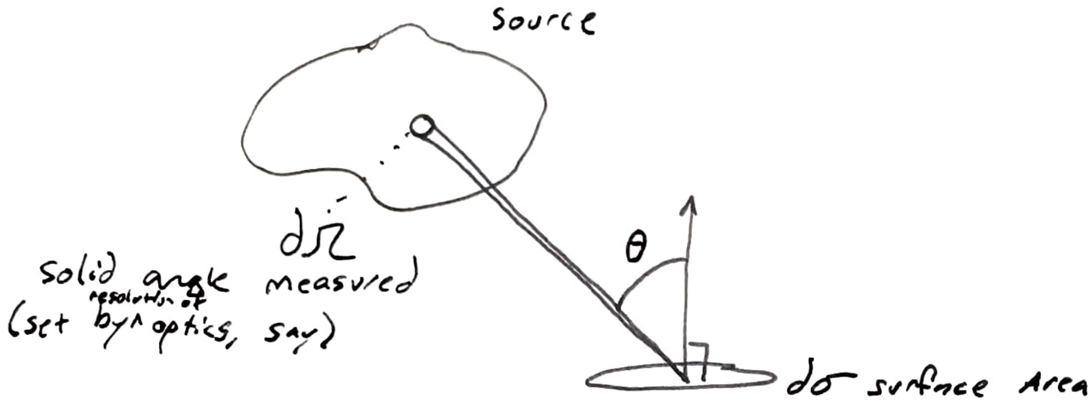
Brightness & Flux density

ALSO Disks w/ Alma

Sources: Condon & Ransom; Taylor, Carilli, & Perley eds Lecture 9, Wrobel + Walker

Brightness/Intensity: I_ν ← per unit frequency

Amount of light per solid angle



Energy hitting ~~det~~ detector in time dt

Def of I_ν → $dE = I_\nu \cos\theta \, d\sigma \, d\Omega \, dt \, d\nu$

- $d\nu$: spectral resolution
- dt: integration time
- $d\Omega$: curvature & filling factor, + baseline length
- $d\sigma$: collecting area

$P = dE/dt$

Property of source we infer: $\boxed{I_\nu} = \dfrac{\boxed{P}}{\boxed{\cos\theta \, d\sigma \, d\nu \, d\Omega}}$ ← What we msr. $\left[\frac{W}{m^2}\frac{1}{Hz}\frac{1}{str}\right]$

Things we think we know about the instrument

The total flux coming from an area, or from an entire ~~discr~~ discrete source. is the Integral over Ω

We rarely integrate over freq, so 'ν' is usually dropped. → $S_\nu = \int I_\nu(\theta,\phi) \cos(\theta) \, d\Omega$

$= \int I_\nu(\theta,\phi) \cos(\theta) \, d\theta \, d\phi \quad \left[\frac{W}{m^2}\frac{1}{Hz}\right]$ or $\left[10^{26} Jy\right]$

S_ν ~~this~~ is ~~used~~ whats reported a catalog

I_ν is whats reported in an image.

The total power of a source

You'd have to surround it to get all the flux.

Measured flux density S_ν

(Area of a sphere surrounding source) × flux assumed to be isotropic

Spectral Luminosity $L_\nu = 4\pi d^2 \cdot S_\nu \quad \left[\frac{W}{Hz}\right]$

If we had a model of emission vs angle eg like a pulsar beam profile,

$$L_\nu = \oint S_\nu(\theta,\phi)\, d\theta\, d\phi$$

If you integrate over all freqs you get

$$L \equiv \int_0^\infty L_\nu \, d\nu \quad [W$$

the bolometric Luminosity.

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Thermodynamics

In free space, Intensity I_ν is conserved

Remember it is a intrinsic property of the source

$$\frac{dI_\nu}{ds} = 0$$

If there is stuff in the way

the probability of absorption dP increases with depth

$$dP \propto ds$$

$$dP = K\,ds$$

the ~~d~~ fractional change in ~~that~~ Intensity

$$\frac{dI}{I} = -dP$$

$$= -K\,ds$$

is negative. Rearrange

$$\frac{dI(s)}{ds} = -K\,I(s)$$

Solve

$$I(s) = e^{-Ks}$$

Integrate from

$$\frac{I(S_{out})}{I(S_{in})} = \exp\left[-\int_{in}^{out} K(s)\,ds\right] \qquad \tau = -\int_{S_0}^{S_{in}} K(s)\,ds$$

$\tau \ll 1$ optically thin

$\tau \gg 1$ optically thick

Emission

Optically thin clouds might emit all along the line of sight

Volume = $ds \cdot d\sigma$

cloud

~~$\dot{P}_{em} \propto$~~

Probability of emission

$$P_{em} \propto \underbrace{ds\,d\sigma}_{\text{volume}}\,\underbrace{d\Omega}_{\text{angle}}\,\underbrace{dt}_{\text{time}}$$

$$\dot{P}_{em} \propto ds\,d\sigma\,d\Omega$$

(~~Recall that~~ cf absorption $\dot{P} \propto ds$)

Emission is per unit area but this (like absorption) scales as depth (s). So we define an emission per line of sight ds

$$j_\nu \equiv \frac{dI_\nu(s)}{ds} \quad \left[\frac{W}{m^3}\frac{1}{Hz}\frac{1}{Str}\right]$$

m^3 ← volume

Add emission + absorption

Day 1 2021

$$\frac{dI_\nu}{ds} = -\kappa I_\nu + j_\nu$$

Recall this is all vs s along line of sight.

- If statistically speaking, all photons emitted are absorbed, things are in thermodynamic equilibrium.
- If no net energy is transferred within
- If the statistical distribution of microstates is consistent with the macrostate (eg a velocity maxwellian distribution @ temp T agreed with a thermometer @ temp T)

→ All cases of Local thermodynamic Equilibrium

See also: The Justice Potter Stewart definition of Pornography

In Thermodynamic Eq two things are true

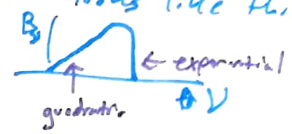
1) No energy gradient

$$\frac{dI}{ds} = 0$$

2) Blackbody spectrum

$$I_\nu = B_\nu(T)$$

Blackbody arises from the number of allowed radiation modes.
More later
It looks like this



Poll for derivation.

$$\frac{dI_\nu}{ds} = 0 = -\kappa B_\nu(T) + j_\nu$$

$$\frac{j_\nu(T)}{\kappa(T)} = B_\nu(T)$$

At low (ie < mm) freqs $kT << h\nu$

Thermal Dispersion ↑ photon energy ↑

$$B_\nu \simeq \frac{2k_B T \nu^2}{c^2}$$

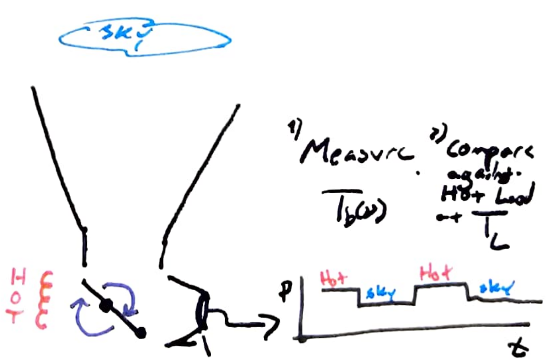
Often if ~~I~~ we! want to measure I_ν which could be some non-Blackbody thing (ie $B_\nu \neq I_\nu$) we might still report the T which would ~~give the me~~ correspond to the thermal power.

ie the Brightness Temp

$$T_b(\nu) \equiv \frac{I_\nu c^2}{2k_B \nu^2}$$

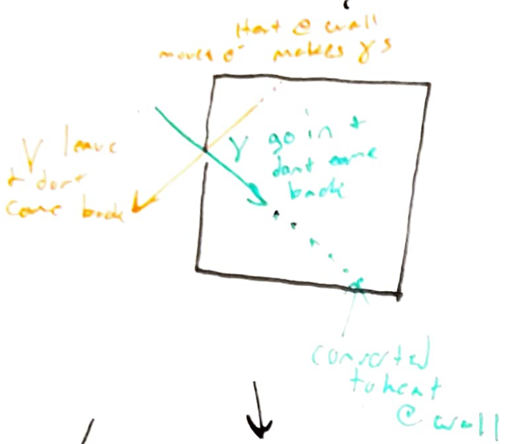
not explicit freq dependence.

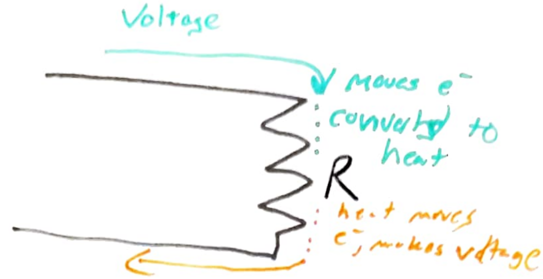
A real BB has the same temp @ all frequencies.



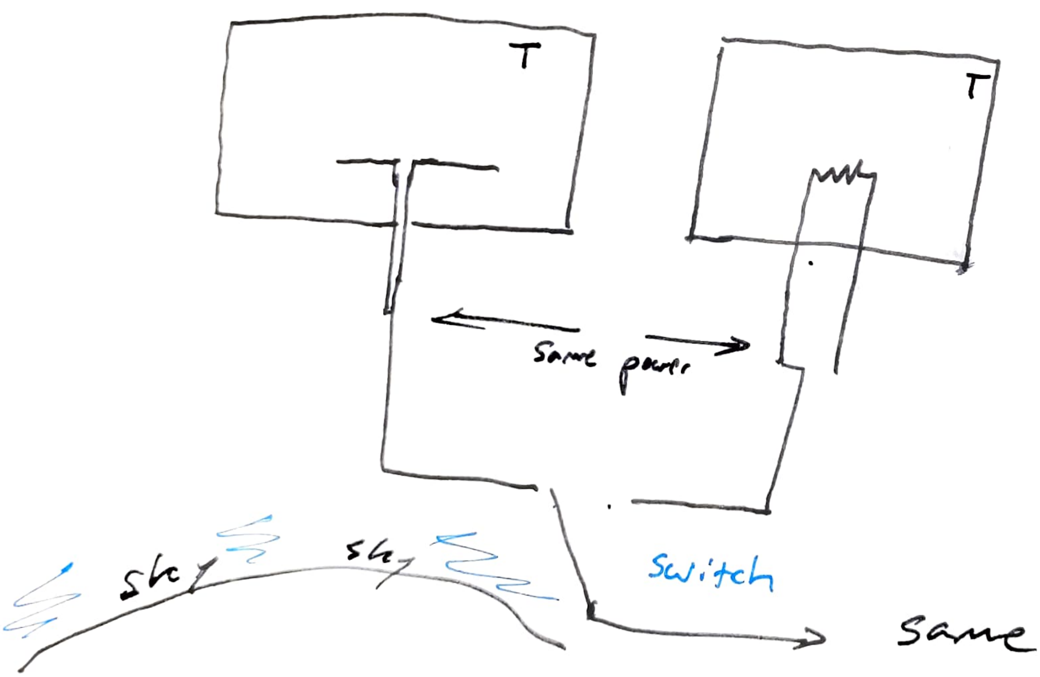
Power from a Resistor

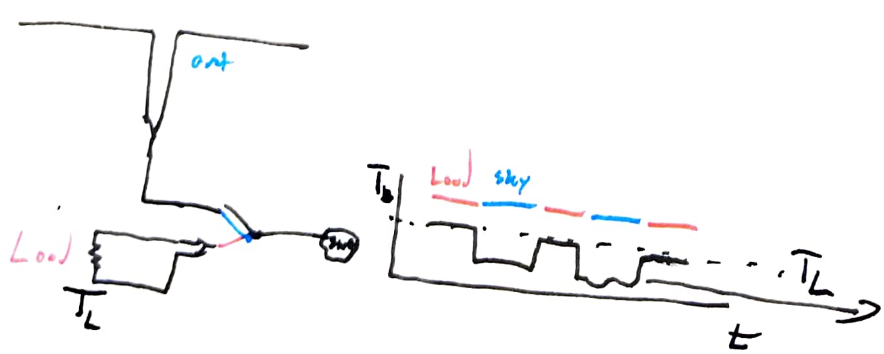
Black Body Cavity





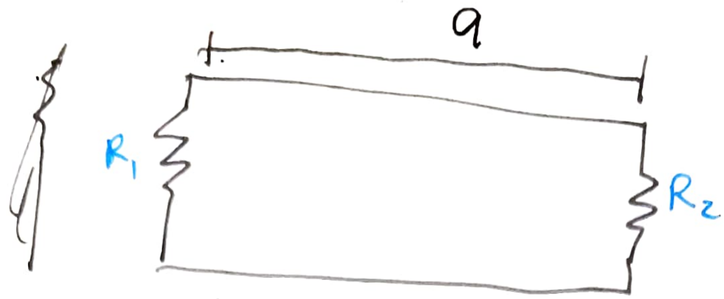
↳ spectrum depends only on temp T





necessarily
Not a real temp!
convert to Intensity

1D Resistor



Modes that fit in a

$$\lambda = \frac{2a}{n}$$

$$V < C \quad = \quad \nu = \frac{V}{\lambda}$$

$$n = \frac{2a}{\lambda}$$

$$= \frac{2a}{V/\nu} = \frac{2a\nu}{V}$$

number of modes per freq.

$$\frac{\partial n}{\partial \nu} = N_\nu = \frac{2a}{V}$$

in Classical TE, each mode has avg energy

$$\langle E \rangle = k_B T$$

Energy per unit freq

$$E_\nu = N_\nu k_B T$$

$$= \frac{2ak_BT}{V}$$

Power in 2 Resistors

$$\boxed{P = \frac{\Delta E_\nu}{\Delta t} = \frac{\Delta E_\nu}{\underbrace{a/V}_{\text{Time to move from R1 to R2}}} = 2k_BT}$$

Power in 1 R

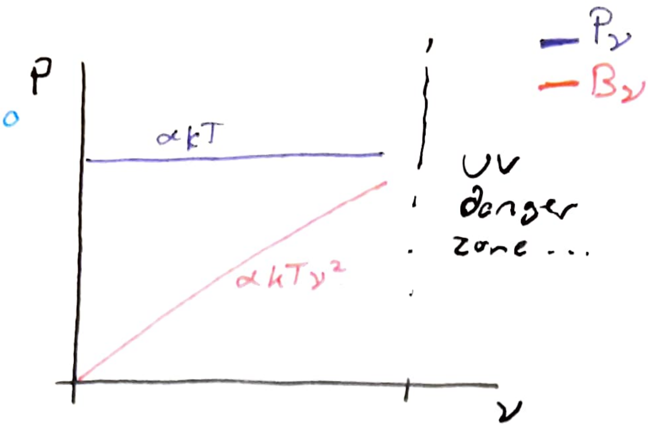
$$\boxed{P = k_BT}$$

Power

$1D \to \nu^0$

$3D \to \nu^2$

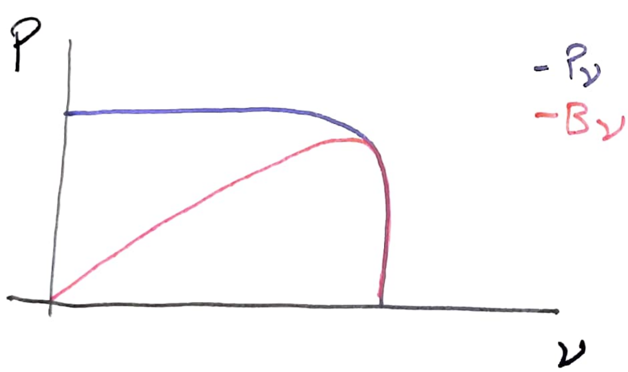
why?



$$\nu_h = \frac{kT}{h}$$

Let $\cancel{h\nu}$ $E_n = nh\nu$ Planck's rule.

$$P_\nu = \frac{h\nu}{\exp(h\nu/kT) - 1}$$



Day 2